Causal Sufficiency

Causal Models and Meanings – Class V

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Plan

1. Causal Sufficiency
   - Sufficiency
   - The presence or absence of determining factors
   - Sufficiency in causal claims
   - Reciprocity and Conjunctive Sufficiency

Plan

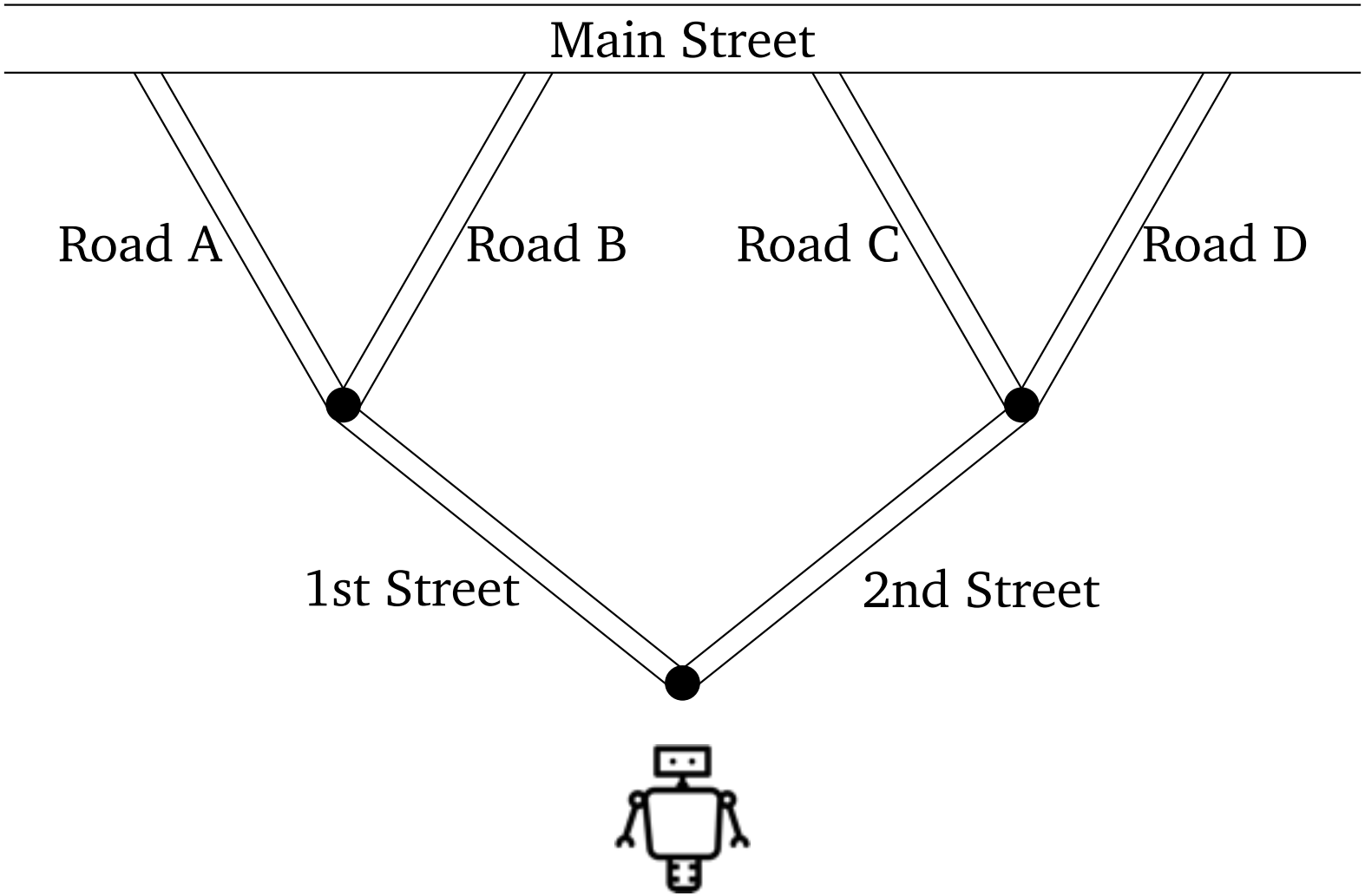
Counterfactual dependence and determinancy

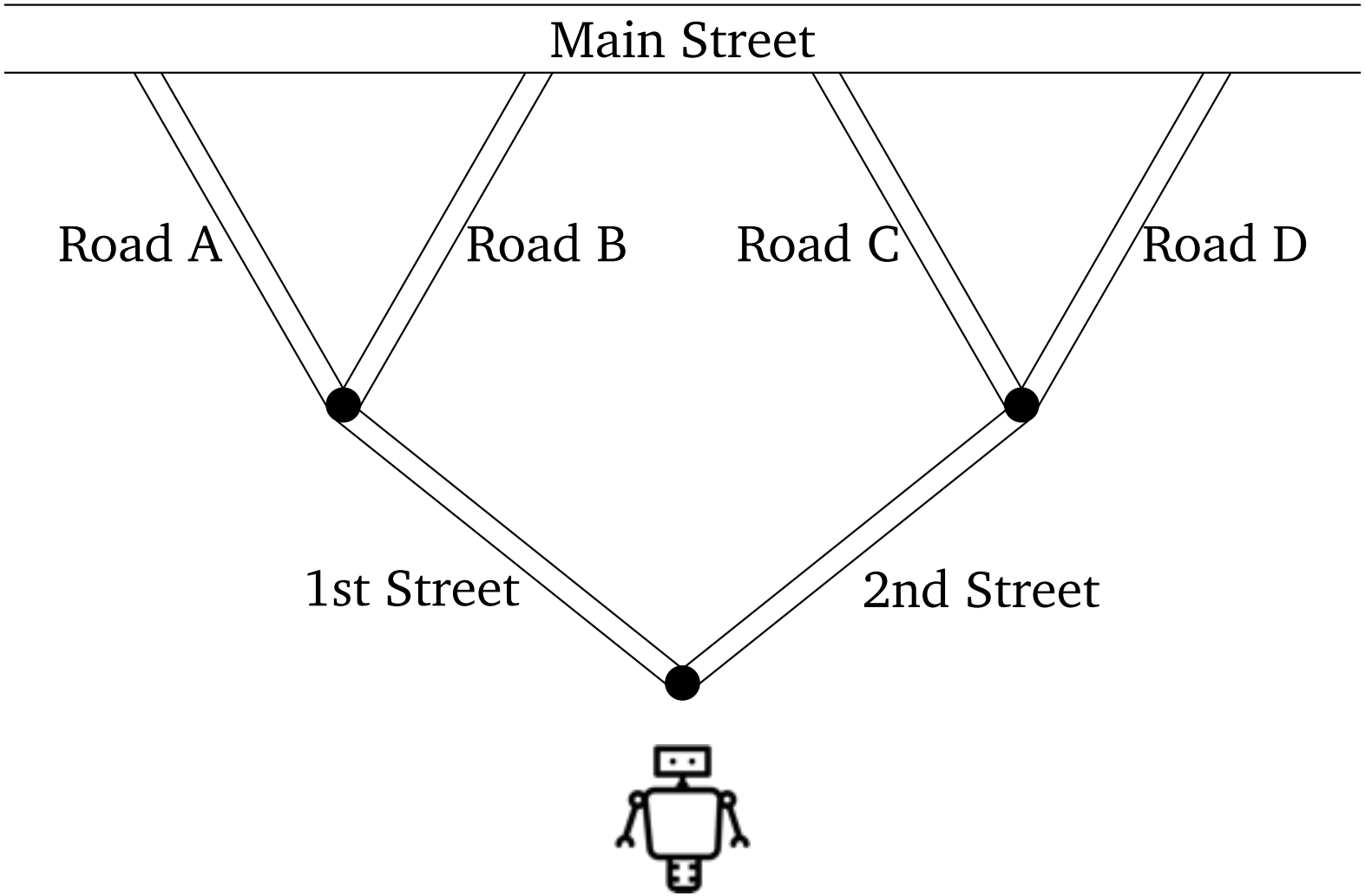
A popular principle:

(**halpernpearl2005causesandexplanations**, Halpern 2016, Beckers 2016, Beckers and Vennekens 2018)

Counterfactual dependence implies actual causation

$$\frac{\text{If } C \text{ had not occurred, } E \text{ would not have occurred}}{C \text{ is an actual cause of } E}$$


Main Street
Road A
Road B
Road C
Road D
1st Street
2nd Street



The robot has to get anywhere on Main Street, choosing between the four ways at random. The robot took 1st Street and then Road B.

(1) The fact that the robot took 1st Street caused it to take Road B

Sufficiency in our intuitive talk about causation

It is often said that causes...

- 'make' their effects happen
- 'produce' them
- 'generate' them
- 'bring' them about

But taking 1st Street did not 'make' the robot take Road B

- Many analyses of causal claims are given in terms of SCMs
- Surprisingly, sufficiency does not play a role in these theories (Halpern 2016, Beckers 2016)
- Though sufficiency is part of some approaches (e.g. Mackie 1965, Baldwin and Neufeld 2004, Braham and van Hees 2012)

Testing the explanation

(1) The fact that the robot took 1st Street caused it to take Road B

Definition (Sufficiency)

One event is sufficient for another just in case in all situations where the first occurs, the second will occur.

Hypothesis 1

(1) is unacceptable because the robot taking 1st Street was not sufficient for it to take Road B

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Causation without sufficiency

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Causation without sufficiency

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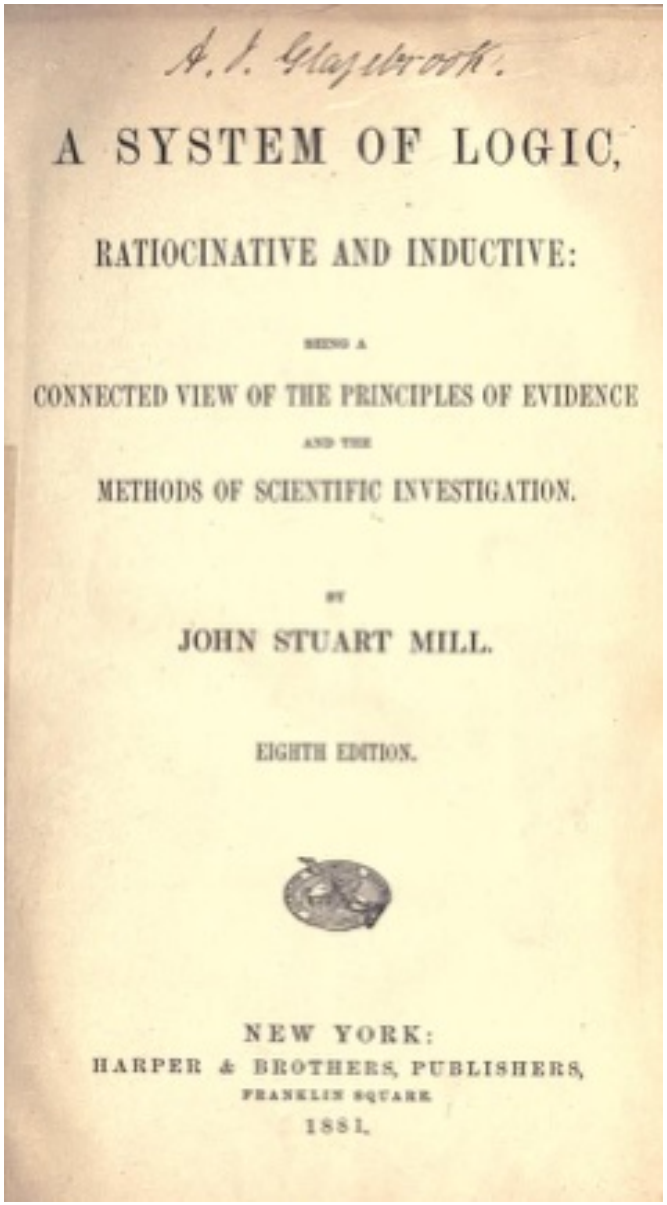
- Flicking the switch was not enough, by itself, to make the light turn on
  - There needed to be power in the building, a working wire from the switch to the light, etc.
- (2) is acceptable nonetheless
- **Intuitive explanation:** (2) is acceptable because we 'assumed' ('took for granted') facts which guarantee that, after flicking the switch, the light will turn on

There is surely a strange confusion of causes and conditions in all this. It may be said, indeed, that without bones and muscles and the other parts of the body I cannot execute my purposes. But to say that I do as I do because of them, [99b] and that this is the way in which mind acts, and not from the choice of the best, is a very careless and idle mode of speaking. I wonder that they cannot distinguish the cause from the condition.

(Plato, *Phaedo* 99a, Benjamin Jowett Translation)

If someone said that without bones and sinews and all such things, I should not be able to do what I decided, he would be right, but surely to say that they are the cause of what I do [...] is to speak very lazily and carelessly. Imagine not being able to distinguish the real cause from that without which the cause would not be able to act as a cause.

(Plato, *Phaedo* 99a, G. M. A. Grube Translation)

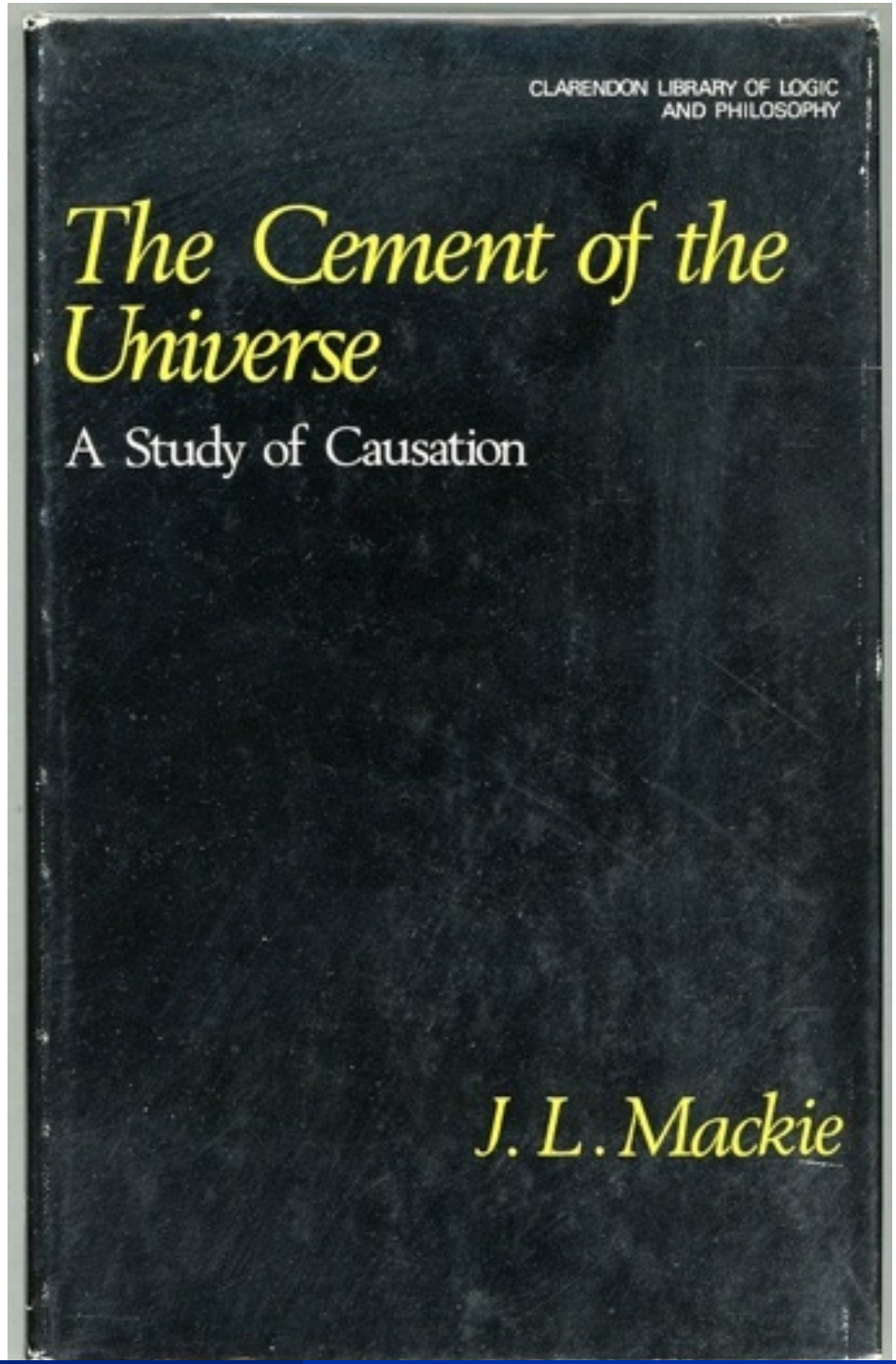


it is very common to single out one only of the antecedents under the denomination of Cause, calling the others merely Conditions. […] The real Cause, is the whole of these antecedents; and we have, philosophically speaking, no right to give the name of cause to one of them, exclusively of the others.

J. S. Mill, *A System of Logic*,
Book III, Chapter V, §3

A cause is: ”what makes the difference in relation to some background or causal field”

Mackie, *The Cement of The Universe*, 1980, p. xi



Enabling conditions

Hypothesis 2

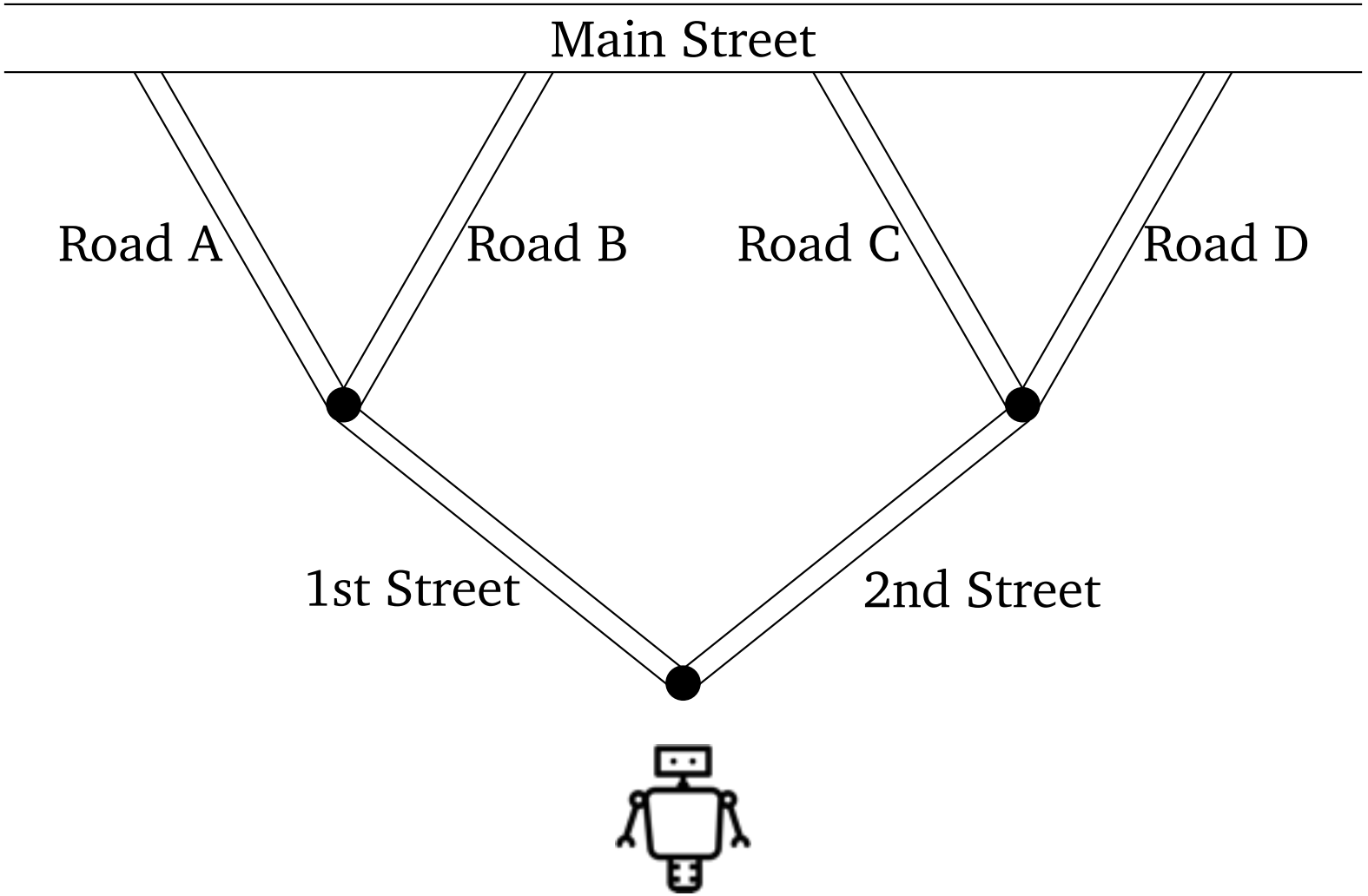
(1) is unacceptable because there was no set of facts such that, within that set, the robot taking 1st Street was sufficient for it to take Road B

Let's test this:

- Change the scenario to provide an enabling condition
- Check whether our judgements also change

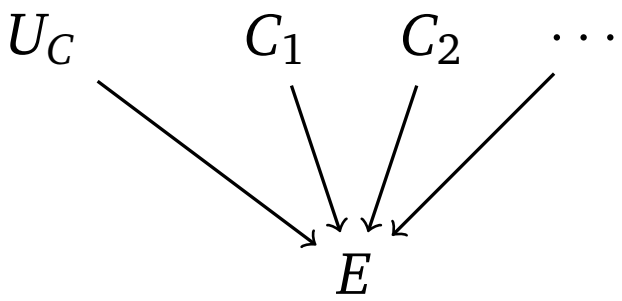
Testing the explanation

Program the robot with the rule: Always change direction!
e.g. if the robot turns left, it must then turn right



Sufficiency: analysis

- Structural causal models are deterministic
- So they are forced to add enabling conditions to represent scenarios that we can intuitively represent as non-deterministic
- In our actual causal reasoning:
  - Sometimes enabling conditions are present
  - Sometimes they are not



$$E = f(C_1, C_2, \dots) \wedge U_C$$

Figure: The error term U_C renders the relationship between E and its parents deterministic

In structural causal models, determining factors are always present

Plan

The need for sufficiency

(3) a. Sue was allowed into the bar because she's over 21.
b. Sue was allowed into the bar because she's over 16.

(4) a. The fact that Sue is over 21 caused the bouncer to let her in.
b. The fact that Sue is over 16 caused the bouncer to let her in.

The need for sufficiency

(5) *The radio spontaneously starts playing music.*

A: Why did the radio turn on?
B: I have no idea. I didn't touch it.
A: I see it's plugged in, and it needs to be plugged in to turn on.
B: Right, but I still have no idea why it started playing.

The need for sufficiency with reasons

Sami and Jan are fun on their own, but always fight when together. A heard that they are both attending a party and therefore decids to skip it.

(6) a. I'm skipping the party for two reasons: because Sami is going and because Jan is going.
b. I'm skipping the party for one reason: because Sami and Jan are going.

(7) a. The reasons why I'm skipping the party are that Sami is going and that Jan is going.
b. The reason why I'm skipping the party is that Sami and Jan are going.

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My intuitive judgement: the (a)-sentences are odd, the (b)-sentences are fine.

The need for sufficiency with reasons

Sami and Jan are each miserable people. Even one of them going to a party is enough to make it a dull event.

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My intuitive judgement: the (a)- and (b)-sentences are both fine.

The sufficiency requirement

- *E because C* $\Rightarrow$ *C* is sufficient for *E*.
- *C cause E* $\Rightarrow$ *C* is sufficient for *E*.

What does it mean for *C* to be sufficient for *E*?

Sufficiency is not logical entailment

(10) a. My laptop turned on because I pushed the power button.
b. Pushing the power button caused the laptop to turn on.

$\not\Rightarrow$ In every logically possible world where I push the power button, the laptop turns on.

These are assertable even though there is a logically possible world where the laptop's battery is empty.

Is C sufficient for E just in case *if* C *would* E is true?

Problem

Many existing semantics of conditionals validate conjunctive sufficiency, predicting that C and E together entail *if* C *would* E.

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Reciprocity implies conjunctive sufficiency

$$\frac{A > B \qquad B > A \qquad B > C}{A > C}\ \text{Reciprocity}$$

$$\frac{A \wedge B}{A > C}\ \text{Conjunctive Sufficiency}$$

Walters and Williams (2013) show that, under mild assumptions, reciprocity also ensures that $A \wedge C$ implies $A > C$.

Consider any true A, C, and any B that is irrelevant to A and C, in the sense that $(B \vee \neg B) > A$ and $(B \vee \neg B) > C$ hold.

$$\frac{A > (B \vee \neg B) \qquad (B \vee \neg B) > A \qquad (B \vee \neg B) > C}{A > C}\ \text{Reciprocity}$$

Given the existence of such a B, Reciprocity tells us that $A \wedge C$ implies $A > C$.

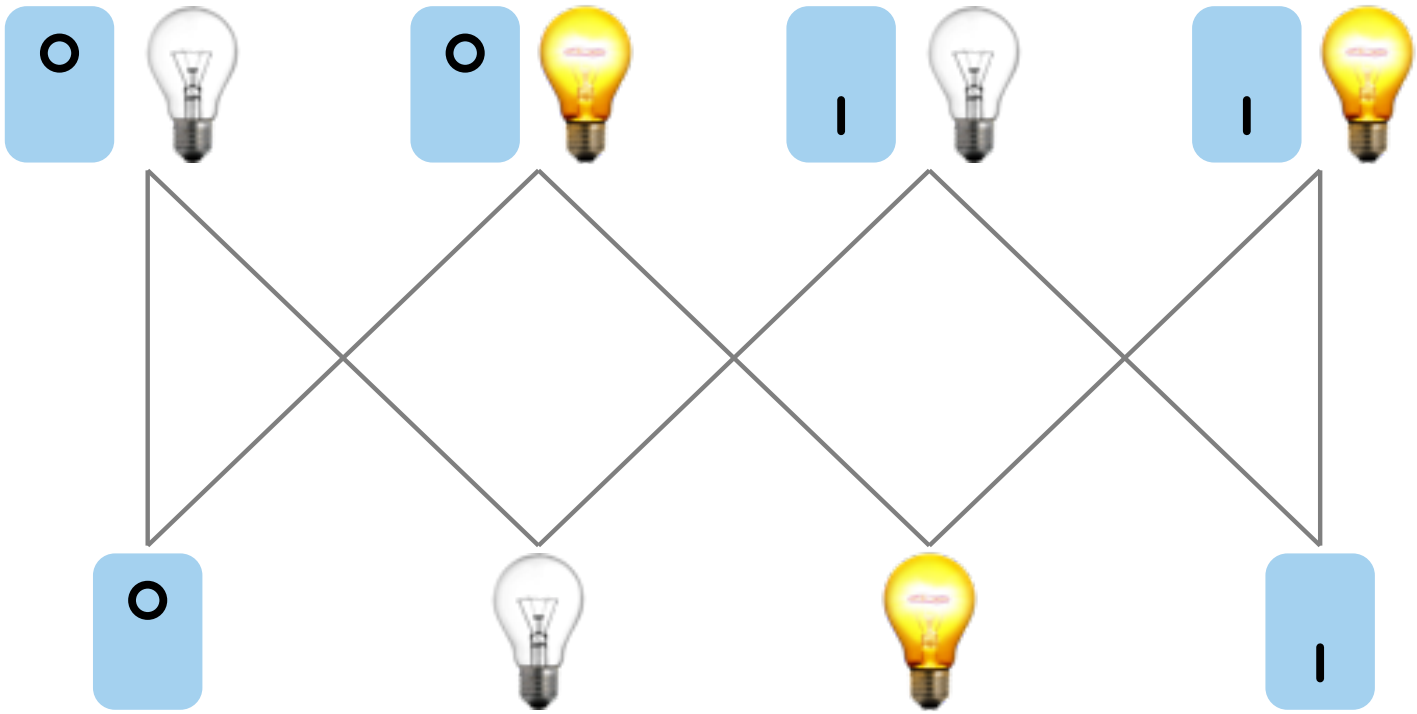


Figure: A state space of the switch and light.

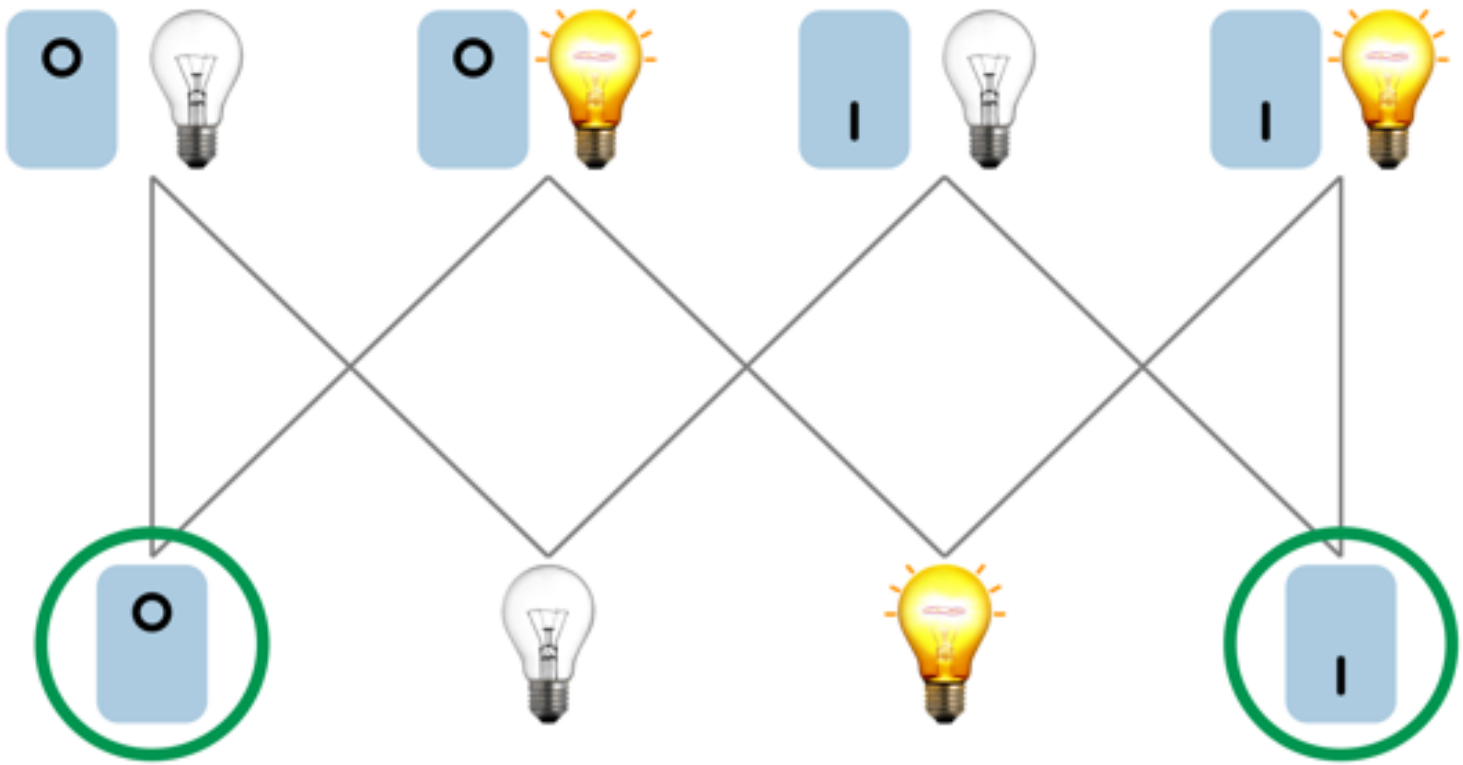
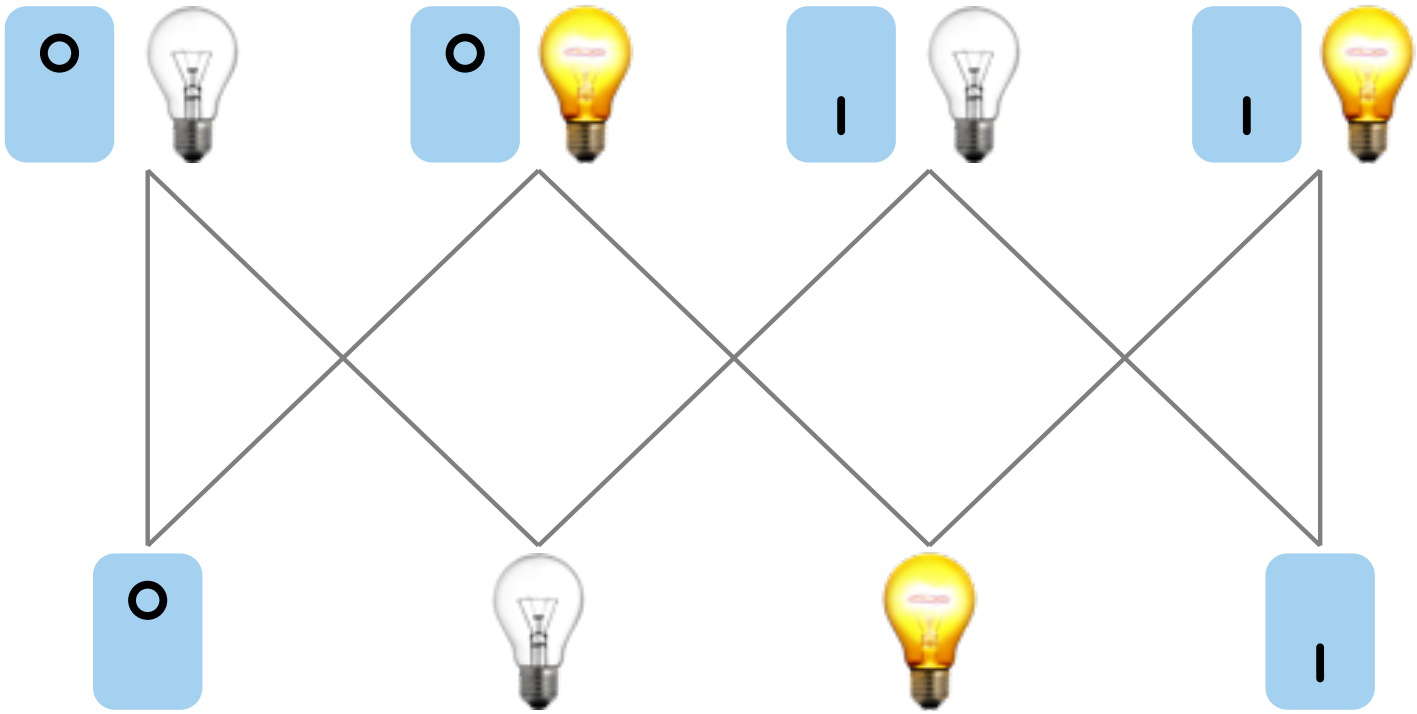


Figure: The states that “the switch is up” is about.



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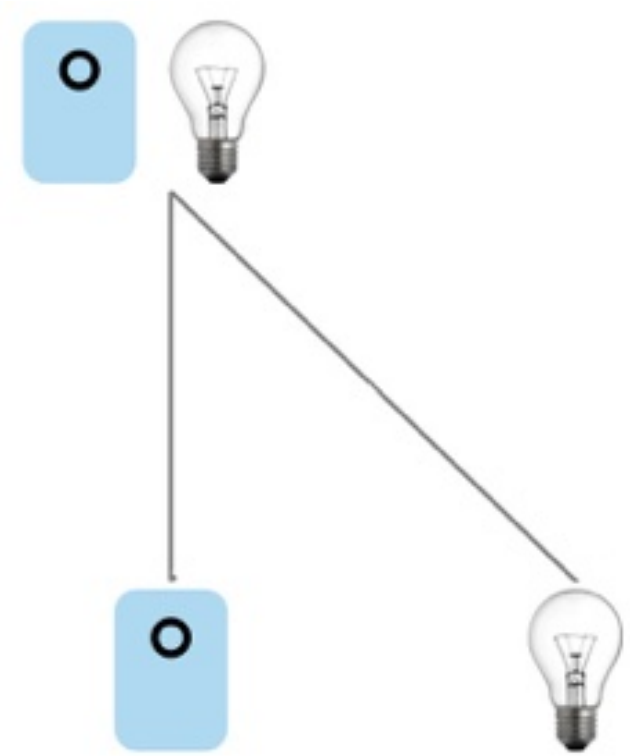

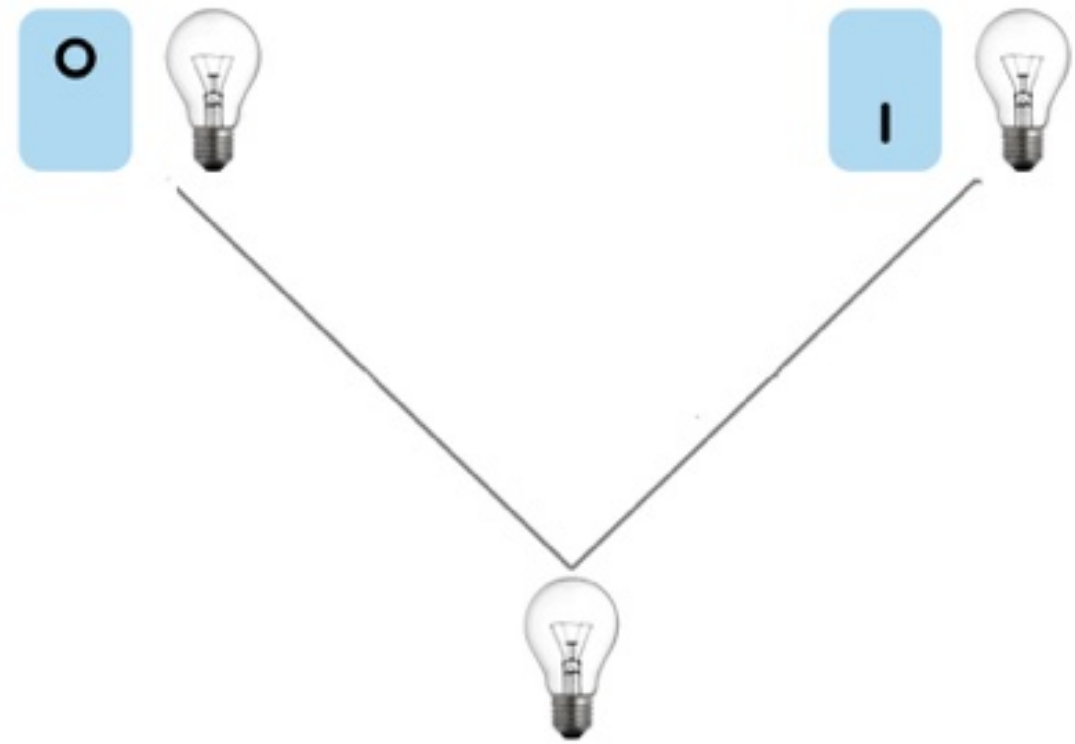
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Analysis of aboutness: a sentence is about the parts of the world that exactly determine its truth value (McHugh 2023, p. 108).

- **The foreground:** the set of states A is about.
- **The background:** the set of states that do not overlap a state in the foreground.

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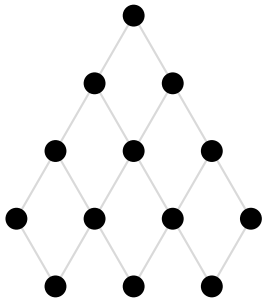
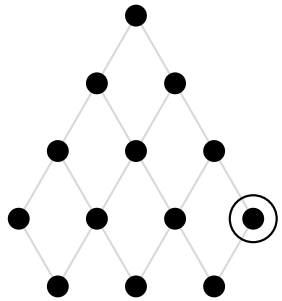
Ceteris paribus

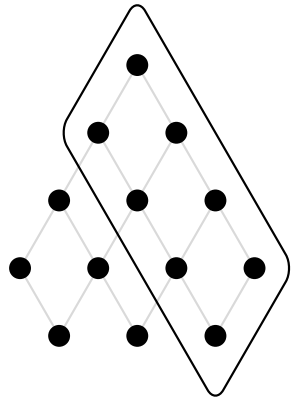
- The background is the *ceteris*, the 'all else' in 'all else being equal'
- *Paribus* means having the *ceteris* as part



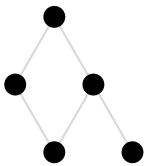
A world w
at a moment in time t



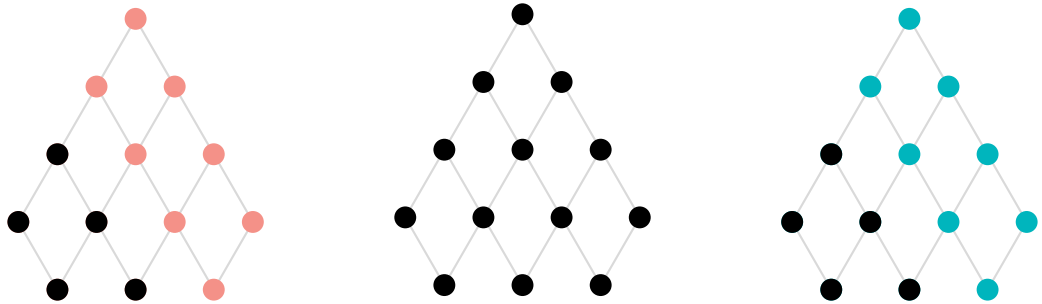
States A is about



Parts of w at t overlapping
a state A is about



Background of A



A-variants of w at t

Figure: Steps to construct the A-variants of a world at a moment in time.

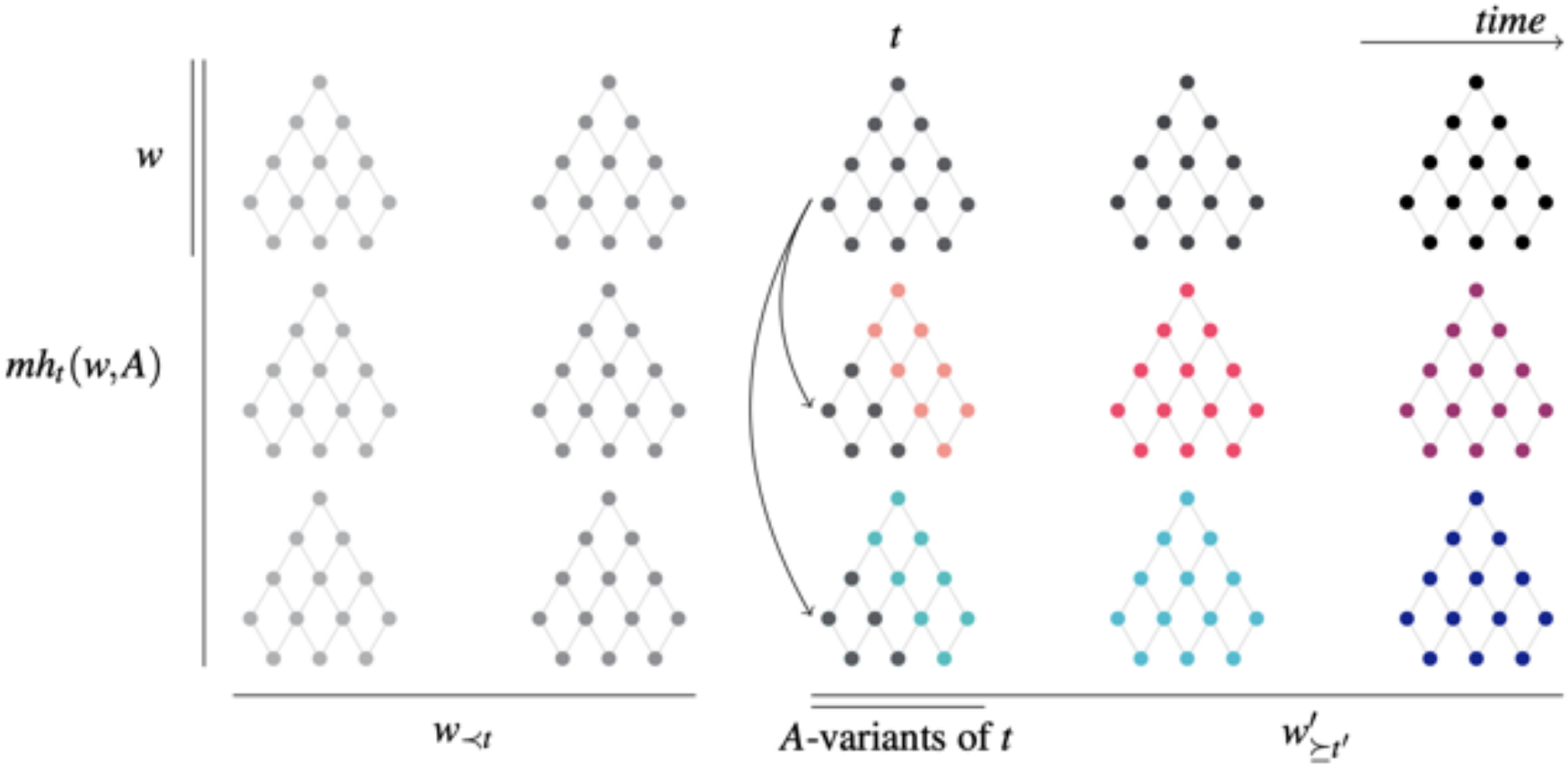


Figure: Constructing the modal horizon.

Infinitely many: a counterexample to Substitution

From Fine (2014, p. 328): There is one poison apple and infinitely many safe apples.

(11) If Eve ate infinitely many of the green apples, she would be fine.

(12) If Eve ate infinitely many of the apples, she would be fine.

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“Eve eats infinitely many of the green apples” is about Eve and the green apples.

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The need for sufficiency

Suppose Sue is 30.

(13) a. Sue was allowed into the bar because she's over 21.
b. Sue was allowed into the bar because she's over 16.

(14) a. The fact that Sue is over 21 caused the bouncer to let her in.
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- "Sue is over 21" and "Sue is over 16" are about Sue's age.

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- We vary Sue's age.

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- "Sue is over 21" and "Sue is over 16" are about Sue's age.
- We vary Sue's age.
- Restrict to those where she is over 21 and 16, respectively.

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- "Sue is over 21" and "Sue is over 16" are about Sue's age.
- We vary Sue's age.
- Restrict to those where she is over 21 and 16, respectively.
- This gives us the diverse ways for Sue to be over 16 (17, 18, ...) beyond merely the actual way in which she is over 16 (being her actual age)

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