Exact Semantics for Permission

Conditional Modality IV

Dean McHugh

Linguistics Department
University of Edinburgh

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Plan

1. **Fine's puzzle.** Two *logically equivalent* sentences, embedded under *may*, permit different things.
2. **Truthmaker semantics.** Fine's response: replace worlds with *states*, and permission with *universal* quantification over verifiers.
3. **Challenge 1: coarseness.** Permission tolerates forbidden ways of complying. (Lecture 1's mixed cases, back again.)
4. **Challenge 2: embedding.** Negation and probability both push the other way.
5. **A positive proposal.** Conditional reduction + an *exact* conditional.
6. **Alternatives and flattening.** Free choice, dual prohibition, probability.

Takeaway

Fine is right that permission needs an exact semantics.
He is wrong about where the exactness lives: not in the permission clause, but in the conditional underneath it.

Recap

The conditional reduction (Anderson 1956, 1958a, Kanger 1957): analyse deontic concepts using a conditional plus an evaluative constant $\textsc{ok}$.

$$O(A) \;\equiv\; \neg A \mathrel{\Box\!\!\rightarrow} \neg\textsc{ok} \qquad\qquad P(A) \;\equiv\; A \mathrel{\Diamond\!\!\rightarrow} \textsc{ok}$$

Weak, not strong. **Mixed cases** showed that $P(A)$ requires only that *some* selected A-world be $\textsc{ok}$, not all.

But free choice seems to want *all*. The resolution:

Permission is weak *within* alternatives, strong *across* them.

Flattening ! collapses alternatives without changing truth conditions, giving an ambiguity that handles probability.

Recap

Sentences denote sets of alternatives; conditionals and modals quantify universally over alternatives (Alonso-Ovalle 2006, Santorio 2018).

Lecture 1's clauses

$$\llbracket A \vee B \rrbracket = \llbracket A \rrbracket \cup \llbracket B \rrbracket \qquad \llbracket \neg A \rrbracket = \{\overline{\bigcup \llbracket A \rrbracket}\}$$

$$\llbracket !A \rrbracket = \{\bigcup \llbracket A \rrbracket\} \qquad \llbracket \Diamond A \rrbracket = \left\{\{w : \forall p \in \llbracket A \rrbracket, f(p,w) \cap \textsc{ok}_w \neq \emptyset\}\right\}$$

Plus a **homogeneity presupposition**: all alternatives have the same status.
This gave dual prohibition, $\neg\Diamond(A \vee B) \models \neg\Diamond A \wedge \neg\Diamond B$.

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Takeaway

Today: all of this was stated over possible worlds. We will see a puzzle that possible worlds *cannot* state — and rebuild the account so that it can.

Part 1

Fine's puzzle

The principle under attack

The orthodoxy: the meaning of a statement is determined by the possible worlds where it is true (Lewis 1970, Partee 1989).

So on the standard theory of modality, *Eve may sing* is true iff some world in the relevant domain is one where Eve sings (Portner 2009:47).

Substitution

If A and B are logically equivalent (true in all the same worlds), then so are $P(A)$ and $P(B)$.

This is not a quirk of one theory. It follows from *any* theory on which the argument of P contributes only a set of worlds — Kripke semantics (Kripke 1959, Hanson 1965), Kratzer (1977), and the account from Lecture 1.

Alternative Eden (Fine 2014a)

Eve finds herself in **Alternative Eden**. There is the Forbidden Fruit a_0, and in addition infinitely many apples $a_1, a_2, a_3, \ldots$

$\cdots \infty$

God tells Eve:

(1) You may eat infinitely many of the apples $a_1, a_2, a_3, \ldots$

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(1) You may eat infinitely many of the apples $a_1, a_2, a_3, \ldots$

(2) You may eat infinitely many of the apples $a_0, a_1, a_2, a_3, \ldots$

(1) does not permit Eve to eat the Forbidden Fruit.
(2) does permit her to eat a_0 alongside infinitely many apples.

Why this is a puzzle

The embedded statements are logically equivalent:

(3a) Eve eats infinitely many of the apples $a_1, a_2, a_3, \ldots$

(3b) Eve eats infinitely many of the apples $a_0, a_1, a_2, a_3, \ldots$

A difference of one cannot separate the finite from the infinite.

If (3a) is true, so is (3b); if (3b) is true, so is (3a).
(Left to right: trivial. Right to left: remove a_0 from an infinite set and it is still infinite.)

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Takeaway

Same set of worlds. Different permissions.
Substitution is false.

The contrast is robust

It does not depend on naming the apples. Suppose a_0 is *red* and the safe apples $a_1, a_2, \ldots$ are *green* (Fine 2014a:329):

(4a) You may eat infinitely many of the green apples.

(4b) You may eat infinitely many of the apples.

Same contrast: only (4b) permits Eve to eat the red apple alongside infinitely many green ones.

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(4b) You may eat infinitely many of the apples.

Same contrast: only (4b) permits Eve to eat the red apple alongside infinitely many green ones.

And the prejacents are again logically equivalent — there are infinitely many green apples, so eating infinitely many apples and eating infinitely many green apples coincide.

Part 2

Fine’s proposal: truthmaker semantics

The diagnosis

Fine (2014a): the puzzle motivates abandoning possible worlds for permission.

The problem is that a set of worlds records whether a sentence is true, but not what makes it true.

(3a) and (3b) are true in the same worlds. But they are made true by *different things*: only (3b) can be made true by a state involving the Forbidden Fruit.

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Takeaway

Replace *worlds* with **states**, and *truth at a world* with **verification by a state** (Fine 2017).

Truthmaker semantics: the ingredients

A **state space** is a set of states S partially ordered by **parthood** $\sqsubseteq$.

- States are *parts* of worlds: the state of it raining, the state of Eve eating a_3. Not complete ways for things to be.
- **Fusion** $s \sqcup t$: the least upper bound of s and t — the smallest state containing both.
- Fine assumes the space is **complete**: every non-empty set of states has a least upper bound. (We will need infinite fusions.)

A **verification** relation between states and sentences, assumed to be exact: when s verifies A, the obtaining of s is *wholly relevant* to A's truth.

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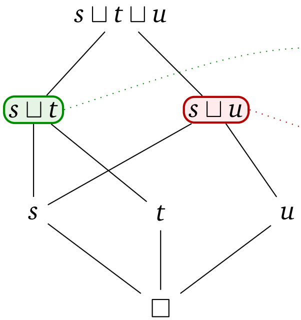
Exactness is the whole point. Truth at a world is monotone: if w makes A true, nothing bigger is at issue. Verification is *not* monotone: s may verify A while $s \sqcup t$ does not.

Exactness, pictured

Let s = Eve eats a_1, t = Eve eats a_2, u = it rains.



$s \sqcup t$ **verifies** *Eve eats a_1 and a_2*

$s \sqcup u$ does **not** verify *Eve eats a_1* —
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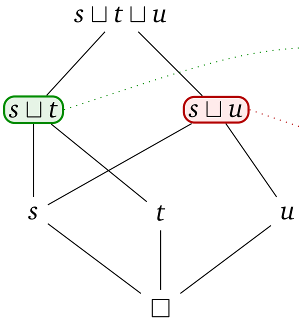

$s \sqcup t$ **verifies** *Eve eats a_1 and a_2*

$s \sqcup u$ does **not** verify *Eve eats a_1* —
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This is what a set of worlds cannot record.
The proposition is the same; the verifiers differ.

Verification clauses

Fine's clauses for the connectives we need:

(6)

a. A state verifies $A \vee B$ iff it verifies A or verifies B.

b. A state verifies $A \wedge B$ iff it is the fusion of a state verifying A and a state verifying B.

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Note what (6a) does *not* say: there is no state that verifies $A \vee B$ "disjunctively", without verifying a disjunct.

Disjunction contributes its disjuncts' verifiers, and nothing else.
Remember this — it does a lot of work in §3.

Modelling Alternative Eden

For each apple a, the state space contains s_a: *Eve eats* a.
Each s_a exactly verifies the atomic sentence p_a.

Fine analyses the two sentences as infinite disjunctions of infinite conjunctions, where $X \subseteq^\infty Y$ means X is an infinite subset of Y:

(7)

Eve eats infinitely many of $a_0, a_1, a_2, \ldots$	$\bigvee_{A \subseteq^\infty \{a_0, a_1, \ldots\}} \bigwedge_{a \in A} p_a$
Eve eats infinitely many of $a_1, a_2, \ldots$	$\bigvee_{A \subseteq^\infty \{a_1, a_2, \ldots\}} \bigwedge_{a \in A} p_a$

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By (6a) and (6b), the verifiers are:

$$\text{with } a_0\text{:} \quad \left\{ \bigsqcup \{s_a : a \in A\} \;:\; A \subseteq^\infty \{a_0, a_1, a_2, \ldots\} \right\}$$
$$\text{without:} \quad \left\{ \bigsqcup \{s_a : a \in A\} \;:\; A \subseteq^\infty \{a_1, a_2, \ldots\} \right\}$$

The first set contains states where Eve eats a_0. The second does not.

Permitted states, and the semantics

Fine adds a property of states: which are **permitted**.

In Alternative Eden the intended assignment is clear:

- every state of Eve eating infinitely many *safe* apples is permitted;
- any state containing s_{a_0} is not.

(8) Descriptive use (Fine 2014a:335)

$P(A)$ is true iff all of the verifiers of A are permitted.

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$P(A)$ is true iff all of the verifiers of A are permitted.

God's pronouncements are not descriptions — they *create* permissions. So Fine adds:

(9) Prescriptive use

$P(A)$ permits all and only those actions which verify A.

On descriptive vs. prescriptive, see Austin (1962), Kamp (1978), Lewis (1979), van Rooij (2000), and Kaufmann (2012).

Fine's solution to the puzzle

In Alternative Eden:

- **(1)** *You may eat infinitely many of* $a_1, a_2, \ldots$
  Every verifier is a fusion of safe-apple states. All permitted. **True ✓**

- **(2)** *You may eat infinitely many of* $a_0, a_1, \ldots$
  Some verifiers contain s_{a_0}. Not permitted. **False ✓**

And on the prescriptive use, only (2) licenses eating a_0 alongside infinitely many apples.

Takeaway

Logically equivalent prejacents; different verifiers; different permissions.
Exactly what the puzzle demanded.

Bonus: free choice falls out

Fine (2018:§6): the account also solves free choice permission (Ross 1941, Kamp 1973).

Derivation. Suppose $P(A \vee B)$ on clause (8): every verifier of $A \vee B$ is permitted.

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So every verifier of A is permitted, and every verifier of B is permitted.

That is $P(A) \wedge P(B)$. $\square$

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Takeaway

Free choice needs no pragmatics, no alternatives, no homogeneity — just a universal clause plus the disjunction clause.

Same on the prescriptive use: being told you may have ice cream or cake permits each of the verifying actions.

Your turn

Compute the verifiers.

Let s_1, s_2, s_3 be the states of Eve eating a_1, a_2, a_3 respectively, with atomic sentences p_1, p_2, p_3.

1. What are the verifiers of $p_1 \vee p_2$?
2. What are the verifiers of $p_1 \wedge (p_2 \vee p_3)$?
3. What are the verifiers of $(p_1 \wedge p_2) \vee (p_1 \wedge p_3)$? Compare with 2.
4. Suppose s_1 is permitted and s_2 is not. On clause (8), is $P(p_1 \vee p_2)$ true? Is $P(p_1)$ true?
5. Does $P(p_1)$ imply $P(p_1 \vee p_2)$ on Fine's account?

Follow-up: item 5 is the crux of the next hour. What would have to change about clause (8) for the answer to flip?

Part 3

Challenge 1: coarse permission

Plan

The principle Fine's clause encodes

Take the **ways** for a statement to obtain to be its verifiers.

(10) Universal Realisability of Permission

A statement is permitted just in case every way for it to obtain is permitted.

Compare Fine's *Universal Realisability of the Antecedent* for counterfactuals: a counterfactual is true when it is true for any way in which its antecedent might be true (Fine 2012:236).

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This is a radical departure. On the standard semantics, $P(A)$ holds iff *some* world in the domain is an A-world that is OK. So permission is **coarse**: a statement can be permitted even when some ways for it to obtain are not.

Is permission coarse?

Five test cases

(11a) There are two switches, A and B. Each can be up or down. The rules specify that A must be up and that B may be in any position.
Are the positions of the switches allowed to agree?

(11b) Alice and Bob are colleagues in Quebec. They know English and French. Bill 96 requires them to speak French at work, but they defy the law by talking to each other at work in English.
Does the new rule permit Alice and Bob to talk to each other at work?

(11c) John's doctor has permitted him to smoke at most seven cigarettes per day.
Does John have permission to smoke more than six cigarettes per day?

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(11c) John's doctor has permitted him to smoke at most seven cigarettes per day.
Does John have permission to smoke more than six cigarettes per day? Yes

(11c) is from Fox (2007:103), who reports the same judgement.

Five test cases, continued

(11d) Alice has many tins of paint in various shades of red, yellow, and blue. Bob asks Alice to paint his house, choosing three paints he likes: Scarlet, Sunflower Yellow, and Yves Klein Blue. He can't decide, so he asks her to use whichever of the three she prefers.
Is Alice allowed to paint Bob's house red?

(11e) Suppose God instead told Eve that she may eat every odd-numbered apple.
Is Eve permitted to eat infinitely many of the apples?

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(11e) Suppose God instead told Eve that she may eat every odd-numbered apple.
Is Eve permitted to eat infinitely many of the apples? Yes

The pattern: something is permitted because *some* way of doing it is, even though other ways are forbidden.

Alice and Bob may talk at work *because they may do so in French*, even though they may not do so in English. John may smoke more than six *because he may smoke seven*.

Coarseness in Alternative Eden

On their *descriptive* use, (1) and (2) are both intuitively true in Alternative Eden.

> *In the particular case of God's permitting Eve to eat infinitely many apples, we may obtain a perfectly good interpretation of the statement by taking it to be true just in case, for some infinite number of the apples, God has in fact granted Eve permission to eat those apples.*
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> *(Fine 2014a:333)*

So Fine would agree: if Eve may eat every odd-numbered apple, it is true that she may eat infinitely many of the apples.

Takeaway

The descriptive use of permission statements is coarse.
But clause (8) is universal. Something has to give.

Plan

But something *is* off about asserting these

The answers in (11) are all ‘Yes’. Yet asserting the corresponding statements outright feels misleading:

(12a) The switches are allowed to agree.

(12b) Alice and Bob are allowed to talk to each other at work.

(12c) John has permission to smoke more than six cigarettes per day.

(12d) Alice is allowed to paint Bob’s house red.

(12e) Eve is permitted to eat infinitely many of the apples.

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(12d) Alice is allowed to paint Bob’s house red.

(12e) Eve is permitted to eat infinitely many of the apples.

They suggest, respectively:

(13a) The switches are allowed to be both up *and* allowed to be both down.

(13b) Alice and Bob are allowed to talk at work in *any* language.

(13c) John may smoke seven, eight, nine, ...

(13d) Alice is allowed to paint the house *any* shade of red.

(13e) For *every* infinite collection of apples, Eve may eat every apple in it.

Call these **universal realisation inferences**: from $P(A)$, we often infer that *every* way for A to obtain is permitted.

Diagnostic 1: sensitive to the question under discussion

Take (12a), *the switches are allowed to agree*.

- Asked the polar question *Are the switches allowed to agree?* — answering Yes is fine.
- Asked *What positions of the switches are allowed?* or *Tell me about the rules governing the switches* — answering *They're allowed to be in the same position* is deeply misleading.

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QUD-sensitivity is a characteristic sign of pragmatic inference (Welker 1994, von Kuppevelt 1996, Breheny, Katsos, and Williams 2006, Zondervan 2009, Degen and Tanenhaus 2015, Ronai and Xiang 2021).

Truth conditions don't shift with the question. Implicatures do.

Diagnostic 2: cancellable

Consider (12d). Asked what Alice may do, Bob may reply:

> *Alice is allowed to paint my house red, yellow, or blue. In fact, she is only allowed to paint it Scarlet, Sunflower Yellow, or Yves Klein Blue.*
> *Fine*

Diagnostic 2: cancellable

Consider (12d). Asked what Alice may do, Bob may reply:

> *Alice is allowed to paint my house red, yellow, or blue. In fact, she is only allowed to paint it Scarlet, Sunflower Yellow, or Yves Klein Blue.*
> *Fine*

If the universal reading were the *literal truth conditions*, this would be equivalent to:

> *Alice is allowed to paint my house any shade of red, yellow, or blue. In fact, she is only allowed to paint it Scarlet, Sunflower Yellow, or Yves Klein Blue.* *Incoherent*

It isn't. So the universal reading is not the literal truth conditions.

Diagnostic 3: metalinguistic negation contour

Denying the inference carries the telltale prosody of denying an implicature:

- *Max didn't talk to Anna OR Robin, he talked to BOTH.*
- *Arseny doesn't LIKE mushrooms, he LOVES them.*
- *I didn't trap two monGEESE, I trapped two monGOOSES.* (Horn 1985)

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The same contour appears in:

- *Alice is not allowed to paint the house RED, she is allowed to paint it SCARLET.*
- *Eve is not permitted to eat infinitely many of the APPLES, she is permitted to eat infinitely many of the GREEN apples.*

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Takeaway

Three diagnostics, one verdict: universal realisation is an implicature, not a truth condition.

It arises on *both* uses — and is especially strong on the prescriptive use, plausibly because prescribing tends to answer *What is permitted?* rather than *Is A permitted?*

Plan

What Fine's clause predicts

The 'Yes' answers in (11) are counterexamples to clause (8), *given* these assumptions on verification:

(15) Assumptions on the verification relation

a. The state of both switches being down verifies that both switches are in the same position.

b. The state of Alice and Bob talking at work in English verifies that they talk to each other at work.

c. The state of John smoking eight cigarettes verifies that he smokes more than six.

d. The state of Alice painting the house crimson verifies that she paints the house red.

e. For every infinite $A \subseteq \{a_0, a_1, \ldots\}$ with $a_0 \in A$, the state of Eve eating every apple in A verifies that she eats infinitely many of $a_0, a_1, \ldots$

These are plausible — and (15e) is explicitly endorsed by Fine.

Given them, Fine's semantics answers 'No' to every question in (11).

Reply 1: contextual restriction

Verifiers may range far beyond what is salient. Fine himself notes:

> *One is allowed to talk in normal social situations, but not too loudly, and to approach other people, but not too closely.* (Fine 2014a:320)

These threaten Universal Realisability too — unless we restrict it:

(16)

a. $P(A)$ is true iff all of the contextually relevant verifiers of A are permitted.

b. $P(A)$ permits all and only those contextually relevant actions which verify A.

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b. $P(A)$ permits all and only those contextually relevant actions which verify A.

Independent support: in a context where someone is known to talk unacceptably loudly, *You are allowed to talk* does sound worse. The restriction is real.

Why contextual restriction doesn't rescue the cases

The switches (11a). Both ways for the switches to agree — both up, both down — are on a par in contextual relevance. Nothing selects one.

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Quebec (11b) is worse.

- *Alice and Bob are allowed to talk to each other at work* is intuitively true.
- They are allowed to do so in French.
- They actually do so in English — which is forbidden.

For contextual restriction to save the day, we would have to declare the English-speaking state contextually *irrelevant*.

But it is the actual way they talk.

If any verifiers of a statement are contextually relevant, its actual ones are.

Reply 2: indeterminate verifiers

The assumptions in (15) assume verifiers are **fully determinate**: an exact verifier of *the ball is red* settles the precise shade.

Fine (2014b:555) calls a state determinate when it verifies a disjunction only if it verifies a disjunct; see Fine (2011) on determinate/determinable in state spaces.

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But exact verification is a primitive. We may set it as we like.

Suppose *both switches down* does *not* verify *the switches agree*. By parity, neither does *both up*. Then what does?

An indeterminate state: one verifying that the switches agree without verifying which position they are in.

Not troubling in itself — we routinely speak of ‘the state of the ball being red’ without settling the shade.

Determinations, and when an indeterminate state is permitted

Associate to each indeterminate state a set of **determinations**.

the switches agree $\rightsquigarrow$ {both up, both down}

(17)

An indeterminate state is permitted just in case at least one of its determinations is permitted.

Determinations, and when an indeterminate state is permitted

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$$\textit{the switches agree} \rightsquigarrow \{\text{both up, both down}\}$$

(17)

An indeterminate state is permitted just in case at least one of its determinations is permitted.

This delivers ‘Yes’ throughout (11). The indeterminate state of the switches agreeing is permitted because *both up* is permitted. ✓

Takeaway

Fine’s account can have it both ways: keep the universal clause (8) over verifiers, while the verifiers themselves are coarse enough that coarseness is respected.

The existential quantifier has simply moved — from the permission clause down into (17).

But: limits on indeterminate states

There are must be limits on which indeterminate states exist. In particular, disjunction must keep Fine's clause (6a).

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Suppose $A \vee B$ had an indeterminate verifier, verifying the disjunction but neither disjunct. With (17):

$$P(A \vee B) \text{ true} \iff P(A) \text{ true or } P(B) \text{ true}$$

Straight into the free choice paradox (Kamp 1973, Merin 1992), which Fine (2018:641) explicitly wants to avoid.

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Straight into the free choice paradox (Kamp 1973, Merin 1992), which Fine (2018:641) explicitly wants to avoid.

Alice is allowed ice cream. Is she allowed *ice cream or to kick her brother*? No.

So we must not have $P(A)$ imply $P(A \vee B)$.

The cleanest version: the tickets

Each customer may buy at most five tickets to the concert.

(18a) Alice may buy more than four tickets. True

(18b) Alice may buy five or more tickets. False

Yet *buys more than four* $\Leftrightarrow$ *buys five or more*. Logically equivalent, differing only in disjunctiveness.

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Fine's account handles this — with the indeterminate-state machinery:

- (18a) is verified by an indeterminate state: she buys more than four, but for no $n \geq 5$ does it verify that she buys n. One determination is *exactly five*, which is permitted. By (17), permitted. ✓
- (18b) is a disjunction. By (6a), its verifiers are those of *buys five* and of *buys more than five*. The latter is an indeterminate state no determination of which is permitted. By (8), false. ✓

Interim summary

	Fine + indeterminate states
Fine's puzzle (Eden)	✓
Coarseness (11a–e)	✓
Free choice	✓
Tickets	✓

We may debate the assumptions on exact verification. But so far Fine survives.

Interim summary

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We may debate the assumptions on exact verification. But so far Fine survives.

The more pressing challenge shows the universal clause is wrong even when we admit indeterminate states.

It comes from **embedding**.

Your turn

Build a coarseness case.

1. Pick a rule system (a game, a workplace policy, a licence, a dietary restriction).
2. Find a description A such that some ways of making A true comply and some do not.
3. Ask a partner: (a) *Is A allowed?* (b) *Under what conditions is A allowed?*
4. Does 'A is allowed' behave differently as an answer to (a) than to (b)?

Follow-up: now try to state what the *indeterminate verifier* of your A would be, and what its determinations are. Is the determination relation obvious, or did you have to make a choice?

Part 4

Challenge 2: embedded permission

Permission under negation

Dual prohibition: *you may not A or B* implies both *you may not A* and *you may not B* (Alonso-Ovalle 2006, Willer 2018, Aloni 2022).

Fine sees the problem, and replies:

> *It might be thought that the present proposal is subject to a difficulty with negative permission statements. [...] However, it is far from clear that the 'not' in the first of these sentences is an external negation, governing the sentence 'you may have candy or ice cream'. Rather, it governs the operator 'may' [...] and the resulting operator 'may not' (or F [for 'Forbidden']) is then subject to a clause similar to the one for permission; 'F[A]' will be true iff all the verifiers of A are not permitted. (Fine 2014a:336)*

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Fair enough for *may not*, which is anyway peculiar — the negation appears below the modal but is interpreted above it (Cormack and Smith 2002, Iatridou and Zeijlstra 2013, Jeretič and Thoms 2023).

Three constructions the reply cannot reach

(19a) You are not allowed to have candy or ice cream.

(19b) No one is allowed to eat *their* candy or *their* ice cream on the bus.
$\neg\exists x P(Fx \vee Gx)$

(19c) I doubt that you are allowed to have candy or ice cream.

In (19b) the negative quantifier *no* must scope above *their* to bind it. In (19c) the negation isn't overt at all — it comes from *doubt*, assuming *doubt that S* $\equiv$ *think that* $\neg S$.

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All three carry the strong inference:

(20a) You are not allowed candy *and* not allowed ice cream.

(20b) No one may eat their candy on the bus, *and* no one may eat their ice cream.

(20c) I think you may not have candy, *and* I think you may not have ice cream.

Fine predicts only the weak inference: at least one is forbidden.

Fine’s metalinguistic negation argument

If we employ what is explicitly an external negation, as in “it is not the case that: you may have candy or ice cream”, then we get the weaker reading, as predicted by the present proposal. (Fine 2014a:336)

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> *If we employ what is explicitly an external negation, as in "it is not the case that: you may have candy or ice cream", then we get the weaker reading, as predicted by the present proposal.* *(Fine 2014a:336)*

But *it is not the case that* need not express literal sentential negation. Horn (1989): even this construction can be metalinguistic.

(21a) It is not the case that John ate SOME of the cookies; he ate ALL of them.

(21b) It is not the case that they had a baby and got married; they got married and had a baby. (based on Wilson 1975:151)

(21c) It is not the case that I LIKE you; I LOVE you.

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Metalinguistic negation targets *any* objectionable feature — inferences, wording, pronunciation. So in Fine's example it may simply be rejecting the free choice inference.

Telling: Fine's weak reading comes through strongest with prosodic focus on *or* — a hallmark of metalinguistic negation.

Permission under probability

It's time for dessert. Alice's parents play a game to decide what dessert she is allowed to have. They roll a die.

1 or 2 $\rightarrow$ fruit (and only fruit)
3 or 4 $\rightarrow$ ice cream (and only ice cream)
5 or 6 $\rightarrow$ cake (and only cake)

The die has just been cast, but we don't know how it landed.

What is the probability that Alice is allowed to have ice cream or cake?

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The die has just been cast, but we don't know how it landed.

What is the probability that Alice is allowed to have ice cream or cake?

Easily accessible answer: two thirds.

Now: what is the probability that she is allowed ice cream *and* allowed cake?

Zero. However the die lands, she gets at most one dessert.

The problem with probability

So in this scenario:

$$\Pr\big(P(A \vee B)\big) \; > \; \Pr\big(P(A) \wedge P(B)\big)$$

But **entailment preserves probability**: if A entails B then $\Pr(A) \leq \Pr(B)$ — a consequence of Kolmogorov's axioms (see Yalcin 2010).

If the probability John is in Berlin is $\frac{2}{3}$, the probability he is in Germany cannot be zero.

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If the probability John is in Berlin is $\frac{2}{3}$, the probability he is in Germany cannot be zero.

Fine's account makes free choice a semantic entailment: $P(A \vee B) \models P(A) \wedge P(B)$. So it requires

$$\Pr\big(P(A \vee B)\big) \;\leq\; \Pr\big(P(A) \wedge P(B)\big)$$

We judged $\frac{2}{3} \leq 0$. Contradiction.

Anticipating an objection: one might read the question as asking for $\Pr(P(A) \vee P(B))$, with wide-scope disjunction. Rephrase: *Alice is allowed to have ice cream or cake* — which all theories, Fine included, parse as $P(A \vee B)$. What is the probability *that sentence* is true? Same answer: two thirds.

A possible repair

Replace the universal clause with its existential analogue:

(22)

$P(A)$ is true iff at least one of the verifiers of A is permitted.

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This fixes both challenges.

- $P(A \vee B) \equiv P(A) \vee P(B)$, so $\neg P(A \vee B)$ implies $\neg P(A)$ and $\neg P(B)$. (19) $\Rightarrow$ (20) ✓
- $\Pr(P(A \vee B)) = \Pr(P(A) \vee P(B)) = \frac{2}{3}$. ✓

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- $\Pr(P(A \vee B)) = \Pr(P(A) \vee P(B)) = \frac{2}{3}$. ✓

And it destroys free choice.

- By (6a), $P(A)$ now implies $P(A \vee B)$. ✗
- *Alice may buy five or more tickets* comes out true when she may buy at most five. ✗

We are stuck between universal and existential.

This is Lecture 1's strong/weak dilemma, in a new framework.

The scoreboard

	Universal (8)	**Existential (22)**
Fine's puzzle (Eden)	✓	✓
Coarseness (with (15) intact)	✗	✓
Coarseness (with indeterminacy)	✓	✓
Free choice / tickets	✓	✗
Dual prohibition (19)–(20)	✗	✓
Probability (dessert)	✗	✓

We want the universal column's free choice row
and the existential column's everything else.

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We want the universal column's free choice row
and the existential column's everything else.

Takeaway

Note what both columns get right: Fine's puzzle. Whatever we build must keep an exact semantics — one that can tell apart logically equivalent sentences.

Part 5

A positive proposal

The starting point: the conditional reduction

From Alan Ross Anderson (1956, 1958a,b) and Stig Kanger (1957); for overviews see Hilpinen (2017) and McNamara and Van De Putte (2022:§3).

Introduce a propositional constant OK: *the relevant deontic ideals are met*.

$$O(A) \equiv \neg A \mathrel{\Box\!\!\rightarrow} \neg \textsc{ok} \qquad\qquad P(A) \equiv A \mathrel{\Diamond\!\!\rightarrow} \textsc{ok}$$

where $\Diamond\!\!\rightarrow$ is the dual of $\Box\!\!\rightarrow$: $A \mathrel{\Diamond\!\!\rightarrow} C \equiv \neg(A \mathrel{\Box\!\!\rightarrow} \neg C)$.

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Why bring this here?

Because — as Fine (2014a:328) himself observes — conditionals give rise to an analogous puzzle.

The same puzzle arises for conditionals

In Alternative Eden, where Eve hasn't eaten any of the apples:

(23a) If Eve ate infinitely many of the apples $a_1, a_2, \ldots$, she would be OK.

(23b) If Eve ate infinitely many of the apples $a_0, a_1, a_2, \ldots$, she would be OK.

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In Alternative Eden, where Eve hasn't eaten any of the apples:

(23a) If Eve ate infinitely many of the apples $a_1, a_2, \ldots$, she would be OK. **True**

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Same logically equivalent antecedents. Same contrast.

Takeaway

This is a strategy, not a coincidence. If we can explain the contrast in (23), the conditional reduction converts that explanation into an account of Fine's puzzle for *permission*.

One solution, two problems.

A simple formalisation

Let f be a contextually supplied **conditional selection function**, taking a sentence and a world (and perhaps a similarity order) and returning a set of worlds: those that result from supposing the sentence true.

(24)

a. $A \mathrel{\Box\!\!\rightarrow} C$ is true at w iff C is true at every world in $f(A, w)$.

b. $A \mathrel{\Diamond\!\!\rightarrow} C$ is true at w iff C is true at some world in $f(A, w)$.

(25) Permission

$P(A)$ is true at w iff some world in $f(A, w)$ is OK.

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(25) Permission

$P(A)$ is true at w iff some world in $f(A, w)$ is OK.

Note the crucial design choice: f takes a *sentence*, not a proposition. That is what buys us exactness — logically equivalent sentences may be sent to different sets of worlds.

Universal realisation, relocated

Recall from §3: universal realisation is an implicature, and on the prescriptive use it is what the utterance *does*.

Both are naturally stated as universal quantification over the selected worlds:

(26)

a. $P(A)$ often carries the implicature that every world in $f(A, w)$ is OK.

b. After an utterance of $P(A)$ on its prescriptive use, every world in $f(A, w)$ is OK.

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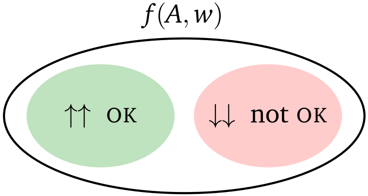
b. After an utterance of $P(A)$ on its prescriptive use, every world in $f(A,w)$ is OK.

The universal quantifier isn't deleted — it's moved out of the truth conditions and into the pragmatics.

Worked example: the switches

Let A = *the switches are in the same position*. A world is OK iff switch A is up. Assume $f(A, w)$ contains both ↑↑-worlds and ↓↓-worlds.



- By (25): *The switches are allowed to be in the same position* is literally true (they may both be up). ✓
- By (26a): unacceptable when it carries the implicature that *every* way is OK — e.g. answering *What positions are allowed?* ✓
- By (26b): used prescriptively, it *makes* both configurations permitted. ✓

Plan

Not the strict or variably strict conditional

Anderson and Kanger take $\Box\!\!\rightarrow$ to be the **strict conditional**, $\Box(A \supset C)$, so $A \Diamond\!\!\rightarrow C \equiv \Diamond(A \wedge C)$.
Hilpinen (1982) suggests the **variably strict** conditional (Stalnaker 1968, Lewis 1973).

Both validate:

(27) Conditional Substitution (a.k.a. LLE, RCEA)

If A and B are logically equivalent, then so are $A \Box\!\!\rightarrow C$ and $B \Box\!\!\rightarrow C$.

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(27) Conditional Substitution (a.k.a. LLE, RCEA)

If A and B are logically equivalent, then so are $A \Box\!\!\rightarrow C$ and $B \Box\!\!\rightarrow C$.

So both predict (23a) and (23b) to be equivalent — and hence, via the reduction, (1) and (2) to be equivalent.

They do not solve Fine's puzzle.

We need a conditional whose selection function is sensitive to *how* the antecedent is stated.

Option 1: Fine's truthmaker counterfactuals (Fine 2012)

Add to the state space a world-relative **transition relation**: when one state is a *possible outcome* of another.

> *A counterfactual $A \boxright C$ is true at w just in case for every verifier t of A and possible outcome u of t at w, u contains a verifier of C.*

Extract a selection function:

(28)

$f_{\mathrm{TM}}(A, w)$ is the set of worlds that contain a possible outcome at w of an exact verifier of A.

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Verifiers as in §2. Transition relation the natural one: any state where Eve eats a_0 leads to a not-OK state; any state where she eats infinitely many without a_0 leads to an OK one.

$f_{\mathrm{TM}}((3\mathrm{b}), w)$ contains a_0-worlds. $f_{\mathrm{TM}}((3\mathrm{a}), w)$ does not. ✓

Option 2: aboutness semantics (McHugh 2022, 2023)

An **aboutness** relation between sentences and states: what parts of the world a sentence is about.

> *A counterfactual $A \mathrel{\Box\!\!\rightarrow} C$ is true at w just in case C is true at every A-world that results from varying what A is about at w and playing the laws forward.*

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$f_A(A, w)$ is the set of A-worlds that result from varying what A is about at w and playing the laws forward.

(3a) is about Eve eating a_1, a_2, ... but not a_0.
(3b) is about Eve eating a_0, a_1, a_2, ...

So interpreting (23a) we *hold fixed* that Eve doesn't eat a_0; interpreting (23b) we let it vary.

$f_A((3b), w)$ contains a_0-worlds. $f_A((3a), w)$ does not. ✓

Why exactness is doing the work

Both theories involve exact relations between states and sentences:

- **Verification** is exact: if s exactly verifies A, no guarantee that a larger state containing s does.
- **Aboutness** is exact: if A is about s, no guarantee that A is about a larger state containing s.

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- **Verification** is exact: if s exactly verifies A, no guarantee that a larger state containing s does.
- **Aboutness** is exact: if A is about s, no guarantee that A is about a larger state containing s.

Plug either into the conditional analysis, and you get an exact semantics of permission — one with the resources to separate logically equivalent statements.

Takeaway

Fine's core claim survives: permission has an exact semantics.
But exactness now enters through the conditional, not through a universal quantifier over verifiers.

That is what lets us keep coarseness.

The score so far

Combined with (25), both conditionals predict (1) and (2) to be literally true — the intuitive result from §3.1. ✓

And in contexts where the QUD is *What is Eve permitted to do?*, (2) carries the false implicature that she may eat a_0 alongside infinitely many apples. ✓

On the prescriptive use, (2) *permits* Eve to eat a_0 alongside infinitely many apples, and (1) does not — exactly the contrast Fine observed. ✓

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But we have bought this with an existential clause.

So free choice is about to come back.

Your turn

Selection functions and exactness.

1. Why exactly is Conditional Substitution (27) a problem for the strict analysis? State the argument in two lines.

Harder: the switches diagram assumed $f(A, w)$ contains both $\uparrow\uparrow$- and $\downarrow\downarrow$-worlds. Is that forced, or an assumption? What would it take to argue for it?

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2. Consider *If Eve ate infinitely many of the green apples, she would be* OK. Which selection function — f_{TM} or f_{A} — makes this true, and why?
3. The paper defines f over *sentences*. What breaks if we define it over propositions (sets of possible worlds) instead?
4. Clause (26a) makes universal realisation an implicature. Sketch how you would derive it as a *quantity* implicature. What is the alternative being negated?

Harder: the switches diagram assumed $f(A, w)$ contains both $\uparrow\uparrow$- and $\downarrow\downarrow$-worlds. Is that forced, or an assumption? What would it take to argue for it?

Part 6

Alternatives and flattening

Free choice, back again

On (25): $P(A)$ says *some* selected A-world is OK; $P(A \vee B)$ says *some* selected $A \vee B$-world is OK.

On a wide variety of selection functions, $f(A, w)$ and $f(A \vee B, w)$ share a world. Whenever they do:

$$P(A) \implies P(A \vee B) \qquad \text{Wrong.}$$

Free choice, back again

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Concretely. Alice may eat fruit but may not eat cake. Supposing *if Alice ate fruit or cake*, among the worlds we consider are some where she eats fruit and not cake. Those are OK.

So *Alice may eat fruit or cake* comes out true.
Incorrect — even on a purely descriptive reading.

Alternatives — with one change

The fix from Lecture 1: conditional antecedents are interpreted via **alternatives** (Alonso-Ovalle 2006, 2009, Ciardelli 2016, Santorio 2018, Willer 2018, Khoo 2022). A conditional is true iff true when supposing *each* alternative.

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Inspiration from alternative semantics (Hamblin 1973, Kratzer and Shimoyama 2002, Alonso-Ovalle 2006) and inquisitive semantics (Ciardelli 2016, Ciardelli, Groenendijk, and Roelofsen 2018).

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Inspiration from alternative semantics (Hamblin 1973, Kratzer and Shimoyama 2002, Alonso-Ovalle 2006) and inquisitive semantics (Ciardelli 2016, Ciardelli, Groenendijk, and Roelofsen 2018).

But: those frameworks make alternatives *sets of worlds*.

So they cannot distinguish God's two sentences — which is the whole problem.

Takeaway

Take the alternatives to be *sentences*, not sets of worlds.

This is exactly in step with (24): f already takes a sentence and a world.

The alternative function

Language $\mathcal{L}$: atomic sentences, flattening !, negation, conjunction, disjunction.

$$A ::= p \mid\, !A \mid \neg A \mid A \wedge B \mid A \vee B$$

(33) $alt : \mathcal{L} \to \mathcal{P}(\mathcal{L})$

$$\begin{aligned}
alt(p) &= \{p\} \\
alt(!A) &= \{A\} \\
alt(\neg A) &= \{\neg A\} \\
alt(A \wedge B) &= \{C \wedge D : C \in alt(A),\ D \in alt(B)\} \\
alt(A \vee B) &= alt(A) \cup alt(B)
\end{aligned}$$

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\end{aligned}$$

Negation always yields a single alternative, as in Lecture 1.
Flattening returns the single alternative A itself — collapsing a disjunction's alternatives without touching its truth conditions.

Eden sentences are atomic

Within this language, represent

Eve eats infinitely many of $a_0, a_1, a_2, \ldots$ and *Eve eats infinitely many of* $a_1, a_2, \ldots$

as atomic sentences. So each has a single alternative.

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as atomic sentences. So each has a single alternative.

This is the point of departure from Fine.

- For Fine, these sentences are infinitary *disjunctions* — which is why the universal clause makes (2) false.
- Here they are atomic — so the existential clause applies, and both come out true.

This is what accounts for the §3.1 judgements: Eve may eat infinitely many of the apples iff there is some infinite set of apples she may eat.

The clauses

Where $|A|$ is the set of worlds where A is true:

(34) Conditionals

a. $A \mathrel{\Box\!\!\rightarrow} C$ is true at w iff for all $B \in alt(A), f(B,w) \subseteq |C|$.

b. $A \mathrel{\Diamond\!\!\rightarrow} C$ is true at w iff for all $B \in alt(A), f(B,w) \cap |C| \neq \emptyset$.

(35) Permission

a. $P(A)$ is true at w iff for all $B \in alt(A), f(B,w) \cap |\textsc{ok}| \neq \emptyset$.

b. After an utterance of $P(A)$ on its prescriptive use, for every $B \in alt(A)$, $f(B,w) \subseteq |\textsc{ok}|$.

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Universal across alternatives, existential within each.

Lecture 1's slogan, now over sentences rather than propositions.

Deriving free choice

Show: $P(A \vee B) \models P(A) \wedge P(B)$.

Suppose $P(A \vee B)$ is true at w. By (35a):

$$\forall C \in alt(A \vee B) : f(C, w) \cap |\textsc{ok}| \neq \emptyset$$

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But $alt(A \vee B) = alt(A) \cup alt(B)$. So:

$$\forall C \in alt(A) : f(C, w) \cap |\textsc{ok}| \neq \emptyset \quad \text{and} \quad \forall D \in alt(B) : f(D, w) \cap |\textsc{ok}| \neq \emptyset$$

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i.e. $P(A)$ and $P(B)$. $\square$

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i.e. $P(A)$ and $P(B)$. $\square$

Takeaway

And the tickets fall out too:
buys more than four is atomic — one alternative, existential clause, true.
buys five or more is disjunctive — every alternative must be permitted, false.

Negation: the homogeneity presupposition

As it stands, (35a) predicts only the *weak* inference:
$\neg P(A \vee B) \Rightarrow \neg P(A) \vee \neg P(B)$. But (19) implies (20).
Same problem for conditionals. Santorio (2018:524): *None of my friends would have fun at the party if Alice or Bob went* implies it of Alice *and* of Bob.

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(36) Homogeneity for conditionals

a. $A \mathrel{\Box\!\!\rightarrow} C$ presupposes that for all $B, B' \in alt(A)$: $f(B,w) \subseteq |C|$ iff $f(B',w) \subseteq |C|$.
b. $A \mathrel{\Diamond\!\!\rightarrow} C$ presupposes that for all $B, B' \in alt(A)$: $f(B,w) \cap |C| \neq \emptyset$ iff $f(B',w) \cap |C| \neq \emptyset$.

(37) Homogeneity for permission

$P(A)$ presupposes that for all $B, B' \in alt(A)$: $f(B,w) \cap |\textsc{ok}| \neq \emptyset$ iff $f(B',w) \cap |\textsc{ok}| \neq \emptyset$.

All or none. Negating ‘all’ now gives ‘none’. ✓

Probability: the last problem

The theory predicts $(A \vee B) \mathrel{\Box\!\!\rightarrow} C \models (A \mathrel{\Box\!\!\rightarrow} C) \wedge (B \mathrel{\Box\!\!\rightarrow} C)$ — so the probability of the former cannot exceed that of either conjunct.

Apparent counterexample. A fair die was thrown and landed 1. Alice says:

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(38) If the die had landed 5 or 6, it would have landed 6.

Probability that what Alice said is true? One half.

(39) If the die had landed 5, it would have landed 6.

Probability? Zero.

Again: a sentence with higher probability than its logical consequence.

Flattening under probability

Proposal: disjunction under probability operators may be parsed as the **flattened** disjunction $!(A \vee B)$ rather than $A \vee B$.
So (38) expresses

$$!(A \vee B) \mathrel{\Box\!\!\rightarrow} B$$

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We choose the parse on which the speaker said something sensible.

And the dessert case. There is a two thirds probability that *Alice is allowed to have ice cream or cake* is true — correctly predicted provided it expresses

$$P!(A \vee B)$$

No free choice inference on the flattened parse. Just: the die landed 3–6.

The final scoreboard

	Fine (8)	**Existential (22)**	**Present account**
Fine's puzzle (Eden)	✓	✓	✓
Coarseness (11a–e)	✗*	✓	✓
Universal realisation infs.	✓	✗	✓(pragmatic)
Free choice / tickets	✓	✗	✓
Dual prohibition	✗	✓	✓(homogeneity)
Probability	✗	✓	✓(flattening)

* recoverable via indeterminate verifiers + principle (17), at the cost of substantial assumptions on exact verification.

Takeaway

A uniform account: the same alternatives-plus-flattening machinery handles conditionals and permission statements alike, and the same exact conditional handles Fine's puzzle in both.

Your turn

Compute the alternatives.

Give $alt(\cdot)$ for each, and say how many alternatives there are.

1. $p \vee q$
2. $\neg(p \vee q)$
3. $!(p \vee q)$
4. $(p \vee q) \wedge r$
5. $!(p \vee q) \wedge r$
6. $\neg\neg(p \vee q)$

Then: for which does $P(\cdot)$ yield a free choice inference?

Compare with Lecture 1. There, $[\![\neg A]\!] = \{\overline{\bigcup [\![A]\!]}\}$ and alternatives were propositions. Here $alt(\neg A) = \{\neg A\}$ and alternatives are sentences. Does item 6 come out the same way in both systems?

Part 7

Conclusion

The argument in five steps

1. **Fine's puzzle.** Two logically equivalent sentences permit different things. Substitution is false, and so is any theory on which P's argument contributes only a set of worlds.
2. **Fine's answer.** Truthmaker semantics: exact verification plus a *universal* clause. It solves the puzzle and delivers free choice.
3. **But permission is coarse.** Fine can absorb this with indeterminate verifiers — but only by relocating an existential quantifier into the determination relation.
4. **And embedding refutes the universal clause outright.** Negation wants existential; probability wants existential; free choice wants universal.
5. **The proposal.** Conditional reduction, an *exact* conditional, and alternatives over *sentences*. Existential within alternatives, universal across them; universal realisation as implicature; flattening for probability.

Where the exactness lives

Fine

exactness in the *permission clause*

$P(A)$ iff *all* verifiers of A permitted

$\longrightarrow$

Present account

exactness in the *conditional*

$P(A) \equiv A \mathrel{\Diamond\!\!\rightarrow} \mathrm{OK}$, with f over sentences

Takeaway

Both are exact: both can separate logically equivalent statements.

Only the second is coarse, and only the second survives embedding.

Open questions

1. **When is flattening available?** We proposed that probability operators license it. Lecture 1 added: when the consequent entails the antecedent. Is there a deeper principle?

2. **The determination relation.** We leaned on an intuitive grasp of when one state determines another. Can it be analysed — and does the analysis vindicate or undermine principle (17)?

3. **Which exact conditional?** f_{TM} and f_{A} both deliver the Eden contrast. Do they come apart elsewhere, and does permission adjudicate?

4. **Atomicity.** We stipulated that the Eden sentences are atomic. What principle decides when an existential-looking sentence raises alternatives? (Cf. event quantification: *it snowed yesterday*.)

5. **First order.** The paper's propositional *alt* extends to quantifiers. Does free choice *any* behave as the parallel with SDA predicts?

Your turn

Some discussion topics:

1. We argued universal realisation is an implicature using three diagnostics. Fine could reply that it is a *presupposition* instead. How would you tell the two apart here?

2. The dessert argument assumes entailment preserves probability. Is there any principled way for a semantic free choice theory to resist that assumption?

3. Making the Eden sentences atomic solves the coarseness problem but looks stipulative. Can you motivate it independently — or find a case where it gives the wrong result?

4. Fine's account and this one agree on Fine's puzzle and disagree on everything else. Design an experiment (or a corpus study) that would distinguish them.

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Thank you

Permission needs an exact semantics.
The exactness belongs in the conditional.

Appendix

Backup slides

An objection: presupposition failure

One might resist: (3a) and (3b) are *not* logically equivalent.

Consider a world w where a_0 does not exist and Eve eats infinitely many of $a_1, a_2, \ldots$

- (3a) is true at w.
- (3b) contains the non-referring term a_0 — so perhaps it is *undefined* at w, not true.

Compare *the King of France is not bald* (Strawson 1950).

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Three replies.

Reply 1: deny the presupposition

Other quantifiers don't carry it. Suppose Eve spoke to Alice and Bob.

Did Eve speak to at least two of Alice, Bob, and the King of France?

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Yes.

Reply 1: deny the presupposition

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Did Eve speak to at least two of Alice, Bob, and the King of France?

Yes.

General principle: a sentence with a non-referring term still gets a truth value *provided its truth value can be settled independently of the problematic referent* (Fodor 1979, Lasersohn 1993, von Fintel 2004, Abrusán and Szendrői 2013).

Lasersohn (1993:116): *The King of France is sitting in this chair* is clearly *false* when we can see the chair is empty; *The King of France is not sitting in this chair* is clearly *true*.

Likewise: knowing Eve ate infinitely many of $a_1, a_2, \ldots$ settles that she ate infinitely many of $a_0, a_1, \ldots$

Replies 2 and 3

Reply 2: legislate the problem away.

(5a) Apple a_0 exists, *and* Eve may eat infinitely many of $a_1, a_2, a_3, \ldots$

(5b) Apple a_0 exists, *and* Eve may eat infinitely many of $a_0, a_1, a_2, a_3, \ldots$

Now both are false where a_0 doesn't exist, so they *are* logically equivalent. The contrast survives.

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Reply 3: retreat to a stronger principle.

- For Kratzer (1977, 2012), $\Diamond A$ and $\Diamond B$ agree whenever A and B agree *throughout the modal base* — weaker than logical equivalence, so a stronger substitution principle.
- Deontic modals are **circumstantial**: the modal base holds fixed that Eve is in Alternative Eden, that a_0 exists, and so on.
- The prejacents agree throughout *that* base. So Kratzer predicts equivalence too.

The puzzle does not turn on a technicality about existence.

Backup: all clauses in one place

Fine's truthmaker semantics

- s verifies $A \vee B$ iff s verifies A or s verifies B
- s verifies $A \wedge B$ iff $s = t \sqcup u$ for some t verifying A, u verifying B
- $P(A)$ true iff all verifiers of A are permitted (descriptive)
- $P(A)$ permits all and only the actions verifying A (prescriptive)

The alternative function

$$
\begin{aligned}
alt(p) &= \{p\} & alt(A \wedge B) &= \{C \wedge D : C \in alt(A), D \in alt(B)\} \\
alt(!A) &= \{A\} & alt(A \vee B) &= alt(A) \cup alt(B) \\
alt(\neg A) &= \{\neg A\}
\end{aligned}
$$

Conditionals and permission (defined only if all alternatives agree in status)

$$
\begin{aligned}
A \mathrel{\Box\!\!\rightarrow} C \text{ true at } w &\iff \forall B \in alt(A) : \ f(B, w) \subseteq |C| \\
A \mathrel{\Diamond\!\!\rightarrow} C \text{ true at } w &\iff \forall B \in alt(A) : \ f(B, w) \cap |C| \neq \emptyset \\
P(A) \text{ true at } w &\iff \forall B \in alt(A) : \ f(B, w) \cap |\textsc{ok}| \neq \emptyset
\end{aligned}
$$

Backup: descriptive vs. prescriptive

Descriptive use. Truly describing the permissions that exist. A speaker informed about the rules but without authority to make them.

Prescriptive use. *Creating* permissions. God in Alternative Eden; a parent telling a child they may have ice cream or cake.

On the distinction see Austin (1962), Kamp (1978), Lewis (1979), van Rooij (2000), and Kaufmann (2012).

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On the distinction see Austin (1962), Kamp (1978), Lewis (1979), van Rooij (2000), and Kaufmann (2012).

Why it matters here.

- Universal realisation inferences arise on *both* uses.
- They are stronger on the prescriptive use.
- Plausible explanation: prescribing tends to answer the open-ended *What is permitted?* rather than the polar *Is A permitted?* — and open-ended questions invite these inferences much more strongly.

So the use-distinction is itself evidence for the pragmatic story.

Backup: why an infinitary language?

Fine analyses the Eden sentences as infinite disjunctions of infinite conjunctions. Those who object to infinitary languages (see Barwise 1981) may instead use a suitably enriched first-order language:

$$\exists A' \subseteq^{\infty} A : \forall x \in A' : \mathit{Eat}(\mathrm{Eve}, x)$$

See Fine (2017:566–69) for a first-order truthmaker semantics.

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But it raises special difficulties:

- Can the domain vary from world to world?
- Do the truthmakers for universally quantified sentences include *totality facts*?

Since these don't bear on the argument, we follow Fine in using infinitary disjunction and conjunction.

Backup: what is OK?

The analysis of OK — of what it is for the deontic ideals to be met — goes beyond the paper. Three options:

1. **Kripke semantics** (Hanson 1965): OK is the set of deontically ideal worlds.
2. **Kratzer** (Kratzer 1981): a world is OK iff it is among the best worlds in the modal base, relative to a deontic preference order.
3. **Options** (Cariani 2013): assume a contextually determined range of options $o_1, o_2, \ldots$, and say w' is OK w.r.t. w iff it is among the deontically best worlds in $f(o_1, w) \cup f(o_2, w) \cup \cdots$

Nothing in the argument depends on the choice.

Backup: contextual relevance

Clause (16) restricted Universal Realisability to *contextually relevant* verifiers. That notion can be spelled out in at least two ways:

- **Circumstantial.** Deontic modals are evaluated holding the current circumstances fixed (Kratzer 1977). Require a statement's contextually relevant verifiers to be compatible with the current circumstances.

- **Options.** Deontic modals are evaluated relative to a range of alternative actions available to the agent (Cariani 2013). Require contextually relevant verifiers to instantiate an available action.

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Neither helps with Quebec. On the first, talking in English *is* compatible with the circumstances — it is what actually happens. On the second, it is plainly an available action.

Backup: solutions to the exercises (sketch)

Part 2 (verifiers).

1. $\{s_1, s_2\}$ — two.
2. $\{s_1 \sqcup s_2,\ s_1 \sqcup s_3\}$ — two.
3. Same as 2. Truthmaker semantics validates distribution here.
4. $P(p_1 \vee p_2)$ is *false* (clause (8): s_2 isn't permitted); $P(p_1)$ is true.
5. No — and that is exactly how Fine blocks the free choice paradox.

Part 5 (selection functions).

1. (3a) and (3b) are logically equivalent; Conditional Substitution says equivalent antecedents give equivalent conditionals; so (23a) and (23b) must agree. They don't.
2. Both. f_{TM}: no verifier of *eats infinitely many green apples* contains s_{a_0}. f_{A}: the sentence isn't about a_0, so we hold fixed that Eve doesn't eat it.
3. Exactness is lost. Logically equivalent sentences are the same proposition, so f must treat them alike — and Fine's puzzle returns.
4. Sketch: the alternative is a stronger statement enumerating the permitted ways; not asserting it implicates the speaker was not in a position to. Hence QUD-sensitivity.

Part 6 (alternatives).

1. $\{p, q\}$ — two
2. $\{\neg(p \vee q)\}$ — one
3. $\{p \vee q\}$ — one
4. $\{p \wedge r,\ q \wedge r\}$ — two
5. $\{(p \vee q) \wedge r\}$ — one
6. $\{\neg\neg(p \vee q)\}$ — one

Free choice for 1 and 4. Item 6: *one* alternative here, since $alt(\neg A) = \{\neg A\}$ flat-out — so double negation kills free choice, just as in Lecture 1's open problem.