Current Accounts of Conditional Permission

Conditional Modality III

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Plan

1. **The conditional reduction.** "*A* is permitted" means something like *if A were true, the deontic ideals would be met*.
2. **Strong vs. weak.** Must *every A*-world we consider be OK, or just *some*?
3. **Mixed cases** say: weak.
4. **But free choice** seems to say: strong.
5. **Alternatives** let us have both.
6. **Flattening** handles counterexamples to SDA and probability.
7. **Extensions**: other modal flavours, *any*, deontic gaps.
8. **Comparisons**: why every existing strong theory fails.

Takeaway

The slogan: permission is weak within alternatives, strong across them.

Part 5

Ambiguity via flattening: counterexamples to SDA, and probability

A worry about semantic free choice

Our account makes free choice a semantic entailment:
$\Diamond(A \vee B) \models \Diamond A \wedge \Diamond B$.

Semantic accounts have to answer for embedded environments. Negation we've handled. But consider **probability operators**.

> *It's time for dessert. Alice's parents roll a die. If it lands 1 or 2, she is allowed fruit (and only fruit). If 3 or 4, ice cream (and only ice cream). If 5 or 6, cake (and only cake). The die has been cast; we don't know how it landed.*
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> *What is the probability that Alice is allowed to have ice cream or cake?*

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Easily accessible answer: two thirds.

Why that's a problem

Probability of *allowed ice cream* ***and*** *allowed cake*? Zero — however the die lands, she gets at most one dessert.

But **entailment preserves probability** (Kolmogorov 1933): if $A \models B$ then $\Pr(A) \leq \Pr(B)$.

If $\Diamond(A \vee B) \models \Diamond A \wedge \Diamond B$, then

$$\Pr\left(\Diamond(A \vee B)\right) \;\leq\; \Pr(\Diamond A \wedge \Diamond B)$$

But we judged $\frac{2}{3} \leq 0$. Contradiction.

Good news: exactly this problem arose for disjunctive antecedents in the conditionals literature, and was solved there. The conditional reduction lets us import the solution.

The ambiguity approach

Take *A or B* to be ambiguous between

$$A \vee B \qquad \text{and} \qquad !(A \vee B)$$

- $A \vee B$ validates SDA (multiple alternatives).
- $!(A \vee B)$ validates it only if $f(p,w) \cup f(q,w) \subseteq f(p \cup q, w)$ — generally, it doesn't.

Ambiguity endorsed by: Loewer (1976), McKay and van Inwagen (1977), Warmbrōd (1981), Hilpinen (1982), Pollock (1984), Alonso-Ovalle (2009), Santorio (2018), Khoo (2021b), and Cariani and Goldstein (2020).

Two independent motivations follow.

Motivation 1: McKay and van Inwagen

(23) If WWII Spain had joined the Allies or the Axis, they would have joined the Axis. Sounds true

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Same antecedent. Opposite verdicts. Ambiguity explains it:

$(23) = !(A \vee B) \mathrel{\Box\!\!\rightarrow} B$	no SDA
$(24) = (A \vee B) \mathrel{\Box\!\!\rightarrow} C$	SDA $\Rightarrow$ predicts falsity

Why a non-ambiguity account struggles

Without ambiguity, the two antecedents mean the same thing.

It is widely assumed that two counterfactual antecedents with the same meaning, in the same context, have the same **counterfactual domain**.

So (23) and (24) share a domain D.

- For (23) to be true, Spain joins the Axis at every $w \in D$.
- At every such w, Hitler is pleased.
- So (24) is true too. Wrong result.

This also casts doubt on pragmatic-context-shift accounts, e.g. Fine's (2012:232) "Suppositional Accommodation", which concerns only whether the antecedent is counterfactually possible — so it can't predict that changing the *consequent* affects SDA.

Motivation 2: conditionals under *probably*

Alice threw a six-sided die. Bob bet the outcome would be 3 or 4. Mary says it landed 2, 3, or 4; Sue says 3, 4, or 5. We have no idea whether the reports are accurate. *(Santorio 2021:633)*

(25) If Mary's report is accurate or Sue's report is accurate, probably Bob won.
Has a true reading

- $(A \vee B) \mathrel{\Box\!\!\rightarrow}$ probably C: *if Mary's is accurate, probably Bob won* $(\frac{2}{3})$ *and if Sue's is accurate, probably Bob won* $(\frac{2}{3})$. True.
- $!(A \vee B) \mathrel{\Box\!\!\rightarrow}$ probably C: *if the die landed between 2 and 5, probably Bob won* — only $\frac{1}{2}$. False.

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(26) If the die landed 2, 3, or 4, probably Bob won. Has a true reading

- Now reversed: !-reading true; unflattened reading false (if it landed 2, Bob certainly lost).

We need both readings. (On *probably*, see Yalcin 2010:928.)

When does each reading appear?

An ambiguity account is only predictive with a generalisation about availability. Proposed:

(27)

If A or B, C may be parsed as $!(A \vee B) \mathrel{\Box\!\!\rightarrow} C$ when C entails $A \vee B$ or contains a probability operator.

Check:

- (23) … *they would have joined the Axis*: consequent entails the antecedent. ✓ Flattening available.
- (24) … *Hitler would have been pleased*: neither condition. ✗ No flattening $\Rightarrow$ SDA $\Rightarrow$ false. ✓
- (25), (26): probability operator. ✓ Both readings available.

Whether (27) follows from deeper principles is open.

Permission under probability, solved

Back to the dessert case.

- **Two thirds** answer = the flattened reading, $\Diamond!(A \vee B)$.
  No free choice inference; just "the die landed 3–6".

- **Zero (or undefined)** answer = the unflattened reading, $\Diamond(A \vee B)$, which entails $\Diamond A \wedge \Diamond B$.
  False (or presupposition failure) however the die lands.

Takeaway

The ambiguity approach licenses both intuitive answers. Semantic free choice survives the probability test.

Worry: could a McKay and van Inwagen-style example flatten $\Diamond(A \vee B)$ to $\Diamond!(A \vee B)$, killing free choice in unembedded contexts? No: by (27) we'd need $\text{OK}_w \subseteq |A|$ or $\text{OK}_w \subseteq |B|$. Since the valuation and OK_w are separate model components, we can always find a model where these fail.

Your turn

Flatten or not?

For each, say which reading generalisation (27) makes available, and what it predicts.

1. If Ann or Bob had come, Bob would have come.
2. If Ann or Bob had come, the party would have been fun.
3. If Ann or Bob had come, it's likely the party would have been fun.
4. You may take the bus or the train.
5. What's the probability that you may take the bus or the train?

Harder: design a scenario that would be a genuine counterexample to (27). What would it have to look like?

Part 6

Extensions

Free choice isn't just deontic

(28a) Mr. X might be in Victoria or in Brixton. (Zimmermann 2000:ex. 6)

(28b) I can lick any man in the house, or drink the lot of you under the table. (Lewis 1977:ex. 4)

(28c) It is legal for you to report this as taxable income or for me to claim you as a dependent. (Lewis 1977:ex. 5)

Epistemic, ability, legal. In each, $\Diamond(A \vee B) \rightsquigarrow \Diamond A \wedge \Diamond B$.

Can the conditional reduction generalise?

Conditionalising the standard theory

Introduce $\Box_0, \Diamond_0, \Box_1, \Diamond_1, \ldots$ with accessibility relations $R_0, R_1, \ldots$ and the standard clauses (Kripke 1959, Hanson 1965, Kratzer 1977)

$$[\![\Box_i A]\!]^s = \{w' : R_i(w') \subseteq [\![A]\!]^s\} \qquad [\![\Diamond_i A]\!]^s = \{w' : R_i(w') \cap [\![A]\!]^s \neq \emptyset\}$$

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Then the two theories collapse:

$$\{w' : R_i(w') \subseteq [\![A]\!]^s\} = \{w' : f_i([\![\neg A]\!]^s, w') \subseteq \textsc{ok}_{i,w'}\}$$
$$\{w' : R_i(w') \cap [\![A]\!]^s \neq \emptyset\} = \{w' : f_i([\![A]\!]^s, w') \cap \textsc{ok}_{i,w'} \neq \emptyset\}$$

But only for the alternative-free version. Unlike the standard theory, the conditional reduction integrates with alternatives: just swap f for f_i in (20)–(21). Result: free choice for arbitrary modal flavours.

First order: free choice *any*

Vendler (1967:79–80): *Take any one of them* "grants you the unrestricted liberty of individual choice".

(31) a. You may take any card.
b. $\Rightarrow$ For each card, you may take it.

(32) a. If you took any card, the rules would be met.
b. $\Rightarrow$ For each card, if you took it, the rules would be met.

Assuming *any* is existential (Horn 1972, Ladusaw 1979):

Propositional	First order
$\Diamond(A \vee B) \Rightarrow \Diamond A \wedge \Diamond B$	$\Diamond\exists x Ax \Rightarrow \forall x \Diamond Ax$
$(A \vee B) \mathrel{\Box\!\!\rightarrow} C \Rightarrow (A \mathrel{\Box\!\!\rightarrow} C) \wedge (B \mathrel{\Box\!\!\rightarrow} C)$	$(\exists x Ax) \mathrel{\Box\!\!\rightarrow} C \Rightarrow \forall x(Ax \mathrel{\Box\!\!\rightarrow} C)$

The parallel between SDA and free choice extends into the first-order domain.

First-order alternative semantics

Models $M = \langle D, I, W, f, \textsc{ok} \rangle$; D the domain, I_w a first-order structure at each w. Assume a constant d' for each $d \in D$.

$$
\begin{aligned}
[\![Ft_1, \ldots, t_n]\!] &= \big\{\{w' : (I_{w'}(t_1), \ldots, I_{w'}(t_n)) \in I_{w'}(F)\}\big\} \\
[\![\forall x A x]\!] &= \{p \cap q \cap \cdots \ : \ p \in [\![Ad'_0]\!],\ q \in [\![Ad'_1]\!],\ \ldots\} \\
[\![\exists x A x]\!] &= \textstyle\bigcup_{d \in D} [\![Ad']\!]
\end{aligned}
$$

Universal quantification = generalised conjunction (pairwise intersection).
Existential quantification = generalised disjunction (union).

Follows Ciardelli, Groenendijk, and Roelofsen (2018:61). With (18) and (21) this predicts $\Diamond \exists x A x \models \forall x \Diamond A x$ and $(\exists x A x) \mathrel{\Box\!\!\rightarrow} C \models \forall x (A x \mathrel{\Box\!\!\rightarrow} C)$.

Flattening in the first-order case

(34a) If anyone ate John's sandwich, he'd be upset. $(\exists x Fx) \mathrel{\Box\!\!\rightarrow} Ga$

(34b) If anyone ate John's sandwich, Bob did. $(!\exists x Fx) \mathrel{\Box\!\!\rightarrow} Fb$

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And under probability, in the dessert scenario:

(35) What is the probability that Alice may eat any of the desserts?

- Answer 1: $\mathrm{Pr}(\Diamond !\exists x Fx)$ — however the die lands she may eat *some* dessert.
- Answer 0: $\mathrm{Pr}(\Diamond \exists x Fx)$ — she never has a *choice*.

Caveat (Embry 2014): not everything expressible with $\exists$ raises alternatives. *It snowed yesterday* $= \exists e(\dots)$ (Davidson 1967, Parsons 1990), but doesn't imply *if any snowing event had happened, I'd have gone skiing* (100 feet of snow: I stay home). Event quantification should be $!\exists$. The existential meaning must be *visible* to the alternative mechanism.

Deontic gaps: Barker's objection

> *"There is a difference between weak permission, which is the absence of prohibition, and strong permission [. . .] which is the assertion that some action is explicitly ok."* *(Barker 2010:12)*

On the **strong** theory, with $F(A) \equiv A \mathrel{\Box\!\!\rightarrow} \neg\textsc{ok}$:
$A \mathrel{\Box\!\!\rightarrow} \textsc{ok}$ and $A \mathrel{\Box\!\!\rightarrow} \neg\textsc{ok}$ can *both* fail. Room for gaps. ✓

On the **weak** theory, it looks as if everything is either permitted or forbidden. Gaps impossible?

Deontic gaps: the fix

Relocate the gap from the *semantics of permission* to the *notion of* OK.

Three classes of worlds — the good, the bad, and the gappy:

- OK: explicitly compatible with the rules
- BAD: explicitly incompatible
- neither: undecided

(36)

$$P(A) \equiv A \mathrel{\Diamond\!\!\rightarrow} \textsc{ok}$$
$$O(A) \equiv \neg A \mathrel{\Box\!\!\rightarrow} \textsc{bad}$$
$$F(A) \equiv A \mathrel{\Box\!\!\rightarrow} \textsc{bad}$$

Whenever the selected A-worlds contain no OK-world but some gappy world, A is neither permitted nor forbidden. Gaps and weak permission coexist.

Two footnotes on gaps

1. Are gaps merely epistemic?
Arguably not. If a justice system has never ruled on whether some behaviour is legal, there is arguably *no fact of the matter* — not merely that we haven't discovered it (see Kress 1989, Madry 1999).

2. Recovering the equivalences.
The tripartite division risks losing $\neg P(A) \equiv O(\neg A) \equiv F(A)$. Recover them where wanted with homogeneity: $P(A)$ and $F(A)$ presuppose every selected A-world is OK or BAD; $O(A)$ presupposes the same for $\neg A$-worlds.

3. Barker's apple-or-pear case is not a gap.
Barker (2010:12): *You may eat an apple or a pear* "neither permits nor forbids eating both". But that's an **implication gap**, not a **deontic gap** — *You may dance* neither permits nor forbids singing, which is perfectly ordinary. Getting a deontic gap requires the extra step that the sentence is neither permitted nor forbidden.

An open problem: double negation

In alternative semantics, $\neg\neg(A \vee B) \neq A \vee B$: the first has one alternative, the second two.

(37a) If Sue hired Alice or Bob, she would hire Alice. $!(A \vee B) \mathrel{\Box\!\!\rightarrow} A$

(37b) Sue must hire neither Alice nor Bob, but she may hire Bob.
$\Box\neg(A \vee B) \wedge \Diamond B$

Intuitively inconsistent. But the account predicts consistency:

- $\Box\neg(A \vee B) \equiv \neg\neg(A \vee B) \mathrel{\Box\!\!\rightarrow} \neg\text{OK} \equiv\ !(A \vee B) \mathrel{\Box\!\!\rightarrow} \neg\text{OK}$
- (37a): every selected $!(A \vee B)$-world is an A-world
- So we only learn A-worlds aren't OK. B-worlds may still be. Hence $\Diamond B$.

Ad hoc patch: keep $[\![\neg A]\!] = \{\bigcup[\![A]\!]\}$ when A isn't a negation, and set $[\![\neg\neg A]\!] = [\![A]\!]$. Cf. analogous moves for anaphora (Gotham 2019, Hofmann 2022, Mandelkern 2022). More elegant solutions welcome.

Part 7

Comparisons

Plan

A problem with strong permission

Intuitively, obligation is also strong:

A statement A is obligated iff no $\neg A$-world is OK $\quad \neg A \mathrel{\Box\!\!\rightarrow} \neg\mathrm{OK}$

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(1) a. You must submit the essay on time.
$A \boxright \neg I$

b. You are not allowed to submit the essay late.
$\neg(A \diamondright I)$ ✓ $\quad \neg(A \boxright I)$ ✗

(2) a. Everyone has to stop at a red light.
$\forall x(Ax \boxright \neg I)$

b. No one is allowed to go through a red light.
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Negation swap is equivalent to Conditional Excluded Middle (CEM):

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Observation

If the underlying conditional $\mathrel{\Box\!\!\rightarrow}$ satisfies CEM, and A is counterfactually possible ($A \mathrel{\Diamond\!\!\rightarrow} \top$), then

- strong and weak permission become equivalent
- and hence both satisfy duality of permission and obligation

Support for CEM

Pair based on Higginbotham (1986):

(3) Everyone will pass if he works hard

(4) No one will fail if he works hard

Failing = not passing

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CEM ensures the equivalence of Higgenbotham's pair

Support for CEM: Probability (see Mandelkern 2018)

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If the probability operator measures the different ways of choosing a single world from the selected worlds, we can predict the intuitive verdict of one half

Wrapping up

- We have evidence that *would* satified CEM
- The $\Box\!\!\rightarrow$ and $\Diamond\!\!\rightarrow$ involved in the conditional analysis of permission should not
  - Otherwise, they would be equivalent: we would lose duality ($\Box A \equiv \neg\Diamond\neg A$) and our account of mixed cases

Upshot: If *would* satifies CEM, *would* does not express the conditional involved in the conditional analysis of deontic logic, $\Box\!\!\rightarrow$

Plan

1. Part 5: Flattening

2. Part 6: Extensions

3. Part 7: Comparisons
   - Does 'would' express $\Box\!\!\rightarrow$?
   - Comparison with alternative theories

4. Part 8: Conclusion

Six strong theories, six conditionals

Strong conditional theory	**Interpretation of "if A, OK"**
Anderson (1967)	Relevant implication (Anderson and Belnap 1962)
von Wright (1968)	A is a "sufficient condition" for OK
Hilpinen (1982) and Nute (1985)	Variably strict conditional (Stalnaker 1968, Lewis 1973)
Asher and Bonevac (2005)	Defeasible conditional of non-monotonic logic (Asher and Morreau 1991, 1995, Morreau 1997)
Lokhorst (1997) and Barker (2010)	Conditional of linear logic (Girard 1987, Avron 1988)
Li (2025)	Dynamic strict conditional (von Fintel 2001, Gillies 2007)

Mixed cases challenge every one of them.

Relevance logic (Anderson 1967)

"*A* is permitted" iff $A \to \text{OK}$, with $\to$ relevant implication in $\mathtt{R}$.

Relevant implication in $\mathtt{R}$ is transitive, hence satisfies antecedent strengthening:

$$A \to B,\ B \to \text{OK} \vdash_{\mathtt{R}} A \to \text{OK}$$

Plausibly *the switches are both down* relevantly implies *the switches are in the same position*: the first guarantees the second, and they are clearly about the same subject matter.

Predicts: "the switches are allowed to agree" $\Rightarrow$ "they are allowed to both be down". Wrong.

Strict and variably strict

von Wright (1968) and Li (2025): $\Diamond A \equiv \Box(A \supset \textsc{ok})$.
Hilpinen (1982) and Nute (1985): the closest A-worlds are OK.

These use the *same* semantic components as natural-language conditionals, so they predict permission statements to pattern with them. Mixed cases show they don't.

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Escape route? Disassociate them — two selection functions:

(38a) $A \mathbin{\Box\!\!\rightarrow} C$ true at w iff $f(|A|, w) \subseteq |C|$

(38b) $\Diamond A$ true at w iff $f'(|A|, w) \subseteq \textsc{ok}_w$

For this to work, f' must be sensitive to the deontic ideals: if the rules require A up, f' must select only $\uparrow\uparrow$; if they require A down, only $\downarrow\downarrow$.

Two ways to make f' rule-sensitive; both fail

Simple: restrict to OK-worlds.

$$f'(p,w) = f(p,w) \cap \textsc{ok}_w$$

Trivialises permission: $f(p,w) \cap \textsc{ok}_w \subseteq \textsc{ok}_w$ always. Everything is permitted.

Two ways to make f' rule-sensitive; both fail

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Sophisticated: *prioritise* OK-worlds.

Modify the similarity order: $u <'_w v$ iff either u is OK and v isn't, or they have the same deontic status and $u <_w v$.

Now consider a world where taking an apple is permitted, taking a pear forbidden. The new order ranks apple-worlds closer than pear-worlds.

Predicts "You may take an apple or a pear" true.

Wrong — and it sacrifices the strong theory's whole selling point, the reduction of free choice to SDA.

Non-monotonic logic (Asher and Bonevac 2005)

$f(A, w)$ returns “the most **normal** A-worlds relative to w”.

> *“If A, then normally B is true at w iff B is true in all normal A-worlds, that is, all A-worlds in which conditions are suitable for assessing (relative to w) what happens when A.”* *(Asher and Bonevac 2005:309)*

But normality isn’t what’s at play. Take the Quebec case:

- Alice and Bob are intuitively permitted to talk at work.
- There is no suggestion that doing so *in French* is more normal — quite the opposite. They habitually speak English.
- So the “normal” worlds are English-speaking worlds, which are forbidden.

Predicts they are forbidden from talking to each other at work. Wrong.

Linear logic (Lokhorst 1997, Barker 2010)

$\Diamond A \equiv A \multimap \text{OK}$, with $\multimap$ linear implication.

Linear implication is a species of material implication:
$A \multimap B \equiv A^{\perp} \parr B \equiv (A \otimes B^{\perp})^{\perp}$,
where $\perp$ is linear negation, $\parr$ multiplicative disjunction, $\otimes$ multiplicative conjunction.

So it inherits classical properties. As Barker (2010:13) observes, A occurs in a downward-entailing position — crucial to his derivations.

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But mixed cases show permission is *not* downward-entailing:

- *The switches are both down* $\models$ *the switches agree*,
  but *allowed to agree* $\not\models$ *allowed to both be down*.
- *A&B talk at work in English* $\models$ *A&B talk at work*,
  but *permitted to talk at work* $\not\models$ *permitted to talk in English*.

Formally (algebraic semantics of Avron 1988): with $\mathcal{A} = \wp(\{\uparrow\uparrow, \uparrow\downarrow, \downarrow\uparrow, \downarrow\downarrow\})$, $\textsc{ok} = \{\uparrow\uparrow, \uparrow\downarrow\}$, “agree” $= \{\uparrow\uparrow, \downarrow\downarrow\}$ and “both down” $= \{\downarrow\downarrow\}$, the two permission statements come out equivalent (both true at $\{\uparrow\uparrow, \uparrow\downarrow, \downarrow\downarrow\}$).

Alternatives from negation? (Schulz 2018, Romoli, Santorio, and Wittenberg 2022)

They propose $\neg(A \wedge B)$ has three alternatives: $A \wedge \neg B$, $\neg A \wedge B$, $\neg A \wedge \neg B$.

Paired with our theory, $\neg\Box(A \wedge B)$ would entail all of $\Diamond(A \wedge \neg B)$, $\Diamond(\neg A \wedge B)$, $\Diamond(\neg A \wedge \neg B)$. Far too strong.

(39) You don't have to read both *Beloved* and *Song of Solomon*, but you do have to read at least one.

$$\neg\Box(A \wedge B) \wedge \Box(A \vee B) \qquad \text{Perfectly consistent}$$

If $\neg\Box(A \wedge B)$ implied $\Diamond(\neg A \wedge \neg B)$, this should be as bad as *You may have ice cream or cake, but you may not have cake*. It isn't.

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$$\neg\Box(A \wedge B) \wedge \Box(A \vee B)$$ Perfectly consistent

If $\neg\Box(A \wedge B)$ implied $\Diamond(\neg A \wedge \neg B)$, this should be as bad as *You may have ice cream or cake, but you may not have cake*. It isn't.

Epistemics behave the same:

(40a) Alice might not have read both books, but she read at least one.

(40b) Out of the two, Alice must have read *Beloved*, but she might not have read both.

Both fine — far better than *Alice might have read Beloved or Song of Solomon, but she didn't read Beloved*.

Part 8

Conclusion

The argument in five steps

1. Many authors think modals and conditionals draw on the same semantic source. Tradition: *A is permitted* $\approx$ *if A, the ideals would be met* — strong.

2. **Mixed cases** refute this. For *A* to be permitted it suffices that supposing *A* returns *some* OK world. And the strong analysis has independent counterexamples (tickets, De Morgan).

3. Weak permission runs straight into **free choice**.

4. **Alternatives** rescue it: permission is weak within each alternative, strong across them. Free choice *and* dual prohibition follow, with homogeneity derived rather than stipulated.

5. **Flattening** gives an ambiguity account handling SDA counterexamples and probability. Generalises to arbitrary modal flavours, to *any*, and to deontic gaps.

Open question: wide scope free choice

The inference from $\Diamond A \vee \Diamond B$ to $\Diamond A \wedge \Diamond B$ (Zimmermann 2000, Geurts 2005, Schulz 2005, Aloni 2022):

(41) You may go to Brixton or you may go to Victoria.

a. $\rightsquigarrow$ You may go to Brixton.

b. $\rightsquigarrow$ You may go to Victoria.

(42) He might be in Brixton or he might be in Victoria.

Open question: wide scope free choice

The inference from $\Diamond A \vee \Diamond B$ to $\Diamond A \wedge \Diamond B$ (Zimmermann 2000, Geurts 2005, Schulz 2005, Aloni 2022):

(41) You may go to Brixton or you may go to Victoria.
  a. $\rightsquigarrow$ You may go to Brixton.
  b. $\rightsquigarrow$ You may go to Victoria.

(42) He might be in Brixton or he might be in Victoria.

If the reduction is right, conditionals should do the same. Remarkably, they do (Wiggins and Edgington 1997, Geurts 2005, Khoo 2021a, McHugh 2025):

(43) If you had gone to Brixton, you would have been fine, or if you had gone to Victoria, you would have been fine.
  a. $\rightsquigarrow$ If you had gone to Brixton, you would have been fine.
  b. $\rightsquigarrow$ If you had gone to Victoria, you would have been fine.

The present theory does not capture this. The parallel between modals and conditionals may run deeper than we have modelled.

Your turn

Discussion — pick one.

1. The three pragmatic diagnostics (QUD-sensitivity, cancellability, metalinguistic negation) are supposed to show the strong inferences are implicatures. How convincing is each? Can you think of a fourth test?

2. Generalisation (27) is stipulated. What deeper principle might it follow from? (Hint: what do a consequent that entails the antecedent and a probability operator have in common?)

3. The double negation problem (§7.4) got an admittedly ad hoc fix. Sketch a more principled alternative.

4. Can you extend the account to *wide scope* free choice? What would have to change?

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Thank you

Appendix

Backup slides

Backup: all semantic clauses in one place

Alternative semantics (propositional)

$$[\![p]\!] = \{V(p)\} \qquad [\![A \wedge B]\!] = \{p \cap q : p \in [\![A]\!], q \in [\![B]\!]\}$$

$$[\![\neg A]\!] = \{\overline{\bigcup [\![A]\!]}\} \qquad [\![A \vee B]\!] = [\![A]\!] \cup [\![B]\!]$$

$$[\![!A]\!] = \{\bigcup [\![A]\!]\}$$

Conditionals (defined only if all alternatives agree in status)

$$[\![A \mathrel{\Box\!\!\rightarrow} C]\!] = \big\{\{w' : \forall p \in [\![A]\!],\ f(p, w') \subseteq |C|\}\big\}$$

$$[\![A \mathrel{\Diamond\!\!\rightarrow} C]\!] = \big\{\{w' : \forall p \in [\![A]\!],\ f(p, w') \cap |C| \neq \emptyset\}\big\}$$

Modals

$$[\![\Box A]\!] = \big\{\{w' : \forall p \in [\![\neg A]\!],\ f(p, w') \subseteq \text{OK}_{w'}\}\big\}$$

$$[\![\Diamond A]\!] = \big\{\{w' : \forall p \in [\![A]\!],\ f(p, w') \cap \text{OK}_{w'} \neq \emptyset\}\big\}$$

Quantifiers

$$[\![\forall x A x]\!] = \{p \cap q \cap \cdots : p \in [\![A d_0']\!], q \in [\![A d_1']\!], \ldots\} \qquad [\![\exists x A x]\!] = \bigcup_{d \in D} [\![A d']\!]$$

Backup: Kratzer's OK

Context supplies a modal base b and ordering source g, both $W \to \wp(\wp(W))$ (Kratzer 1977).

$$w' \leq_{g(w)} w'' \text{ iff } \forall p \in g(w):\ w'' \in p \Rightarrow w' \in p$$
$$w' <_{g(w)} w'' \text{ iff } w' \leq_{g(w)} w'' \text{ but not } w'' \leq_{g(w)} w'$$
$$\textsc{best}(b, g, w) = \{w' \in \bigcap b(w) : \neg\exists w'' \in \bigcap b(w):\ w'' <_{g(w)} w'\}$$

Set $\textsc{ok}_w = \textsc{best}(b, g, w)$.

Note: the tripartite OK/BAD/gap division of §7.3 goes beyond this, which only divides worlds in two. Deriving a tripartite division inside a Kratzerian framework is an open question.

Backup: alternative vs. inquisitive semantics

Nothing today hinges on the choice. One could equally use inquisitive semantics (Ciardelli 2016, Ciardelli, Groenendijk, and Roelofsen 2018).

The one divergence: **Hurford disjunctions**, $A \vee (A \wedge B)$, where one disjunct entails the other. The two frameworks assign different alternatives (Ciardelli and Roelofsen 2017).

Hurford's constraint (Hurford 1974) predicts these to be odd anyway, unless the weaker disjunct is exhaustified: $A \vee (A \wedge B) \to (A \wedge \neg B) \vee (A \wedge B)$. Since they are independently odd, we set them aside.

Backup: why the subjunctive, in more detail

The security guard, required to stay at his post, can felicitously say:

- *I am required to be at my post right now.*
- *I'm not allowed to be elsewhere right now.*

Subjunctive reduction: *If I were not at my post right now, the rules would be broken.* Fine.

Indicative reduction: *If I am not at my post right now, the rules are/will be broken.* Odd.

Why? Indicatives are restricted to worlds compatible with the common ground (Stalnaker 1975). Everyone knows he's at his post.

The "right now" matters: it blocks a generic reading. Without it, *If I am not at my post, the rules will be broken* is acceptable even when he is at his post.

Backup: solutions to the exercises (sketch)

Part 4 exercise

1. $[\![p \vee q]\!] = \{|p|, |q|\}$ — two
2. $[\![\neg(p \vee q)]\!] = \{\overline{|p| \cup |q|}\}$ — one
3. $[\![\neg p \wedge \neg q]\!] = \{\overline{|p|} \cap \overline{|q|}\}$ — one (same as 2)
4. $[\![(p \vee q) \wedge r]\!] = \{|p| \cap |r|,\ |q| \cap |r|\}$ — two
5. $[\![!(p \vee q) \wedge r]\!] = \{(|p| \cup |q|) \cap |r|\}$ — one
6. $[\![\neg\neg(p \vee q)]\!] = \{|p| \cup |q|\}$ — one (the trap: not equivalent to 1 in alternative semantics; see §7.4)

Free choice arises for 1 and 4.

Part 5 exercise

1. Consequent entails antecedent ⇒ flattening available ⇒ can be true.
2. Neither condition ⇒ no flattening ⇒ SDA ⇒ entails both simplifications.
3. Probability operator ⇒ both readings available.
4. Unembedded ⇒ unflattened ⇒ free choice.
5. Probability operator ⇒ ambiguous: high (flattened) vs. possibly zero (unflattened).