Angelika Kratzer's Theories of Modals and Conditionals

Conditionals: Meaning and Applications

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Plan

1. Deontic paradoxes
2. Premise semantics
3. The restrictor view
   - Adverbs of quantification
   - Implementing the restrictor view

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Deontic paradoxes (Kratzer 1981)

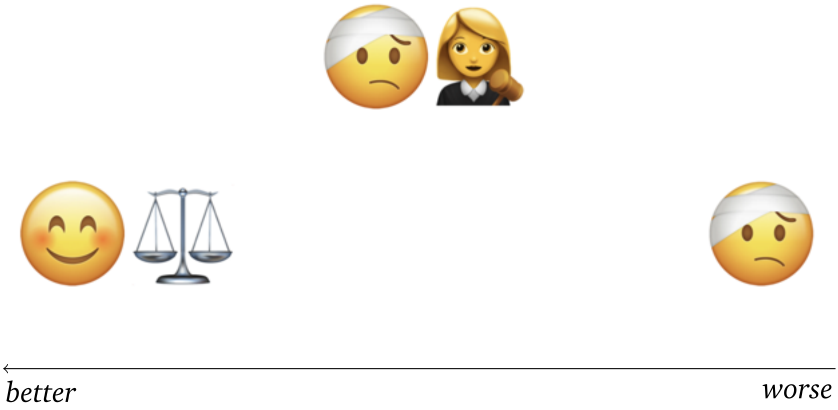
(1) a. Justice must be given to everyone.
b. If someone was unjustly treated, the injustice must be amended.
c. If someone was unjustly treated, the injustice must be rewarded.

Assumptions

1. $\Box A$ is true at w iff A is true at every v accessible from w (Kripke 1959)
2. v is accessible from w iff the rules of w are followed in v
3. *If A, must B* expresses $\Box(A \rightarrow B)$

Problem

$\Box A$ entails $\Box(\neg A \rightarrow B)$ and $\Box(\neg A \rightarrow \neg B)$ (Kratzer 1981, p. 70)


better
worse

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Kratzer's premise semantics (Kratzer 1981)

A proposition is a set of worlds. Let f and g be functions, each taking a world and returning a set of propositions.

f is the *modal base*. It determines the accessibility relation.

A world w' is accessible from a world w just in case w' makes true every proposition in the modal base at w.

$$wRw' \quad \equiv \quad w' \in \bigcap f(w)$$

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- If $f(w)$ is empty, every world is accessible.
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- For bouletic modals (*want*, *desire*), the modal base is doxastic: it only contains worlds compatible with the agent's beliefs at w.

Kratzer's premise semantics (Kratzer 1981)

g is the *ordering source*.

It determines an order over worlds.

One world w' is as good as another w'', relative to $g(w)$, just in case every proposition in $g(w)$ that is true at w'' is true at w'.

$$w' \leq_{g(w)} w'' \quad \equiv \quad \forall p \in g(w) : w'' \in p \to w' \in p$$

Kratzer's premise semantics (Kratzer 1981)

- g is realistic just in case w makes every proposition in $g(w)$ true: $w \in g(w)$

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Kratzer (1981): counterfactuals have a totally realistic ordering source

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Lewis (1975) on adverbs of quantification

always, usually, mostly, rarely, sometimes, never...

(2) a. Sometimes, if a cat is happy, it purrs.
  b. Never, if a cat is happy, does it purr.
  c. Usually, if a cat is happy, does it purr.

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Sometimes/never/usually is this conditional sentence true: "if a cat is happy, it purrs".

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Sometimes/never/usually is this conditional sentence true: "if a cat is happy, it purrs".

But they *can* be parsed as $Q(A)(C)$, where Q is a binary operator

- $\textit{sometimes}(A)(C) = A \cap C \neq \emptyset$
- $\textit{never}(A)(C) = A \subseteq \neg C$
- $\textit{usually}(A)(C) = \#(A \wedge C)$ is greater than $\#(A \wedge \neg C)$

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The restrictor view

The history of the conditional is the story of a syntactic mistake. There is no two-place if … then connective in the logical forms for natural languages. If-clauses are devices for restricting the domains of operators. Whenever there is no explicit operator, we have to posit one.

(Kratzer 1986, p. 656)

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Kratzer's implementation of the restrictor view

Conditional antecedents add the proposition they express to the modal base.

if A, C is true with respect to w, f, g just in case

C is true with respect to $w, f + A, g$,

where $f + A(w) = f(w) \cup \{|A|\}$ for any world w, and $|A|$ is the set of worlds where A is true

Kratzer on the restrictor view

> *quantificational operators, including adverbs of quantification, probability operators, and other modal operators, do not operate over "conditional propositions." The persistent belief that there could be such "conditional propositions" is based on a simple syntactic mistake. If-clauses need to be parsed as adverbial modifiers that restrict operators that might be silent and a distance away. This is what we might call "the restrictor view" of if-clauses.*

(Kratzer 2012, p. 107)

ANGELIKA KRATZER

AN INVESTIGATION OF THE LUMPS OF THOUGHT

Contents

Kratzer derives the ordering source from a set of propositions

$$w' \leq_{g(w)} w'' \quad \equiv \quad \forall p \in g(w) : w'' \in p \rightarrow w' \in p$$

Example

Illustration: p and q are true at w.
Consider: "if p were false, q would still be true"

$$g_1(w) = \{p, q\}$$
$$g_2(w) = \{p \cap q\}$$

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