The Truth Conditions of Conditionals

Conditionals: Meaning and Applications

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Plan

1. The truth conditions of counterfactuals
   - The material conditional
   - The strict conditional
   - Goodman's cotenability theory
   - Stalnaker's semantics
   - Lewis's Semantics
   - Kratzer's semantics

2. Does 'would' express necessity?

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Material conditional

$A \supset C$ is true just in case A is false or C is true: $\neg A \vee C$

Some observations:

- Counterfactuals usually have false antecedents
- If counterfactuals expressed the material conditional, then all counterfactauls with a false antecedent would be...

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- Counterfactuals usually have false antecedents
- If counterfactuals expressed the material conditional, then all counterfactauls with a false antecedent would be...
  trivially true

(1) If the lights were on, the room would be dark.

(2) If the lights were on, the moon would explode.

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Strict conditional

Going back to C. I. Lewis *A Survey of Symbolic Logic* (1918)

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Kripke semantics: $\Box A$ is true at a world w just in case A is true at every world accessible from w.

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Kripke semantics: $\Box A$ is true at a world w just in case A is true at every world accessible from w.

Taking $A > C$ as $\Box(A \supset C)$ validates:

1. **Antecedent strengthening.** $A > C$ implies $(A \wedge B) > C$
2. **Contraposition.** $A > C$ implies $\neg C > \neg A$
3. **Transitivity.** $A > B$ and $B > C$ imply $A > C$

Counterexamples to Antecedent Strengthening

Antecedent strengthening. $A > C$ implies $(A \wedge B) > C$

Goodman's match (Goodman 1947)

(3) If I struck this match, it would light. $S > L$

(4) If I dipped this match in water and struck it, it would light. $(W \wedge S) > L$

Counterexamples to Antecedent Strengthening

From Lewis (1973a)

(5) If kangaroos had no tails, they would topple over.

Counterexamples to Antecedent Strengthening

From Lewis (1973a)

(5) If kangaroos had no tails, they would topple over.

(6) If kangaroos had no tails but used crutches, they would not topple over.

Counterexamples to Contraposition

Contraposition. $A > C$ implies $\neg C > \neg A$

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(7) a. If it rained, it didn't rain hard.
    b. If it rained hard, it didn't rain.

(von Fintel and Heim 2011)

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(von Fintel and Heim 2011)

(8) a. (Even) if Goethe hadn't died in 1832, he would still be dead now.
b. If Goethe were alive now, he would have died in 1832.

(due to Angelika Kratzer)

Counterexamples to Transitivity

Transitivity. $A > B$ and $B > C$ imply $A > C$

(9) a. If Hoover had been a Communist, he would have been a traitor.
b. If Hoover had been born in Russia, he would have been a Communist.
c. If Hoover had been born in Russia, he would have been a traitor.

(Stalnaker 1968, p. 38)

Plan

Goodman's cotenability theory (Goodman 1947)

> *"A is cotenable with D, ... if it is not the case that D would not be true if A were":* $\neg(A > \neg D)$

$A > B$ is true iff

(i) there is a finite set of true statements D cotenable with A such that A and D together with some laws imply B, and

(ii) there is no set of true statements H cotenable with A such that A and H together with some laws imply $\neg B$.

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> *"[T]o establish any counterfactual, it seems that we first have to determine the truth of another. If so, we can never explain a counterfactual except in terms of others, so that the problem of counterfactuals must remain unsolved. Though unwilling to accept this conclusion, I do not at present see any way of meeting the difficulty."*

Plan

Stalnaker's semantics

Consider a possible world in which A is true, and which otherwise differs minimally from the actual world. 'If A, then B' is true (false) just in case B is true (false) in that possible world.

(Stalnaker 1968, pp. 33–34)

Stalnaker's semantics

Let W be the set of possible worlds, R an accessibility relation, and λ the 'absurd world' where every proposition is true.

Let f be a conditional selection function, taking a sentence and a world and outputting a world, $f : \mathcal{L} \times W \to W$.

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Constraints on the selection function:

1. A is true at $f(A, w)$.
2. $f(A, w) = \lambda$ is the absurd world λ only if there is no possible world with respect to w in which A is true.
3. If A is true in w then $f(A, w) = w$.
4. If A is true in $f(B, w)$ and B is true in $f(A, w)$, then $f(A, w) = f(B, w)$.

   These conditions on the selection function are necessary in order that this account be recognizable as an explication of the conditional

(Stalnaker 1968, p. 36)

Stalnaker's semantics

The informal truth conditions that were suggested above required that the world selected differ minimally from the actual world. [...] the selection is based on an ordering of possible worlds with respect to their resemblance to the based world.

(Stalnaker 1968, pp. 35–36)

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Lewis's sphere semantics (Lewis 1973b)

A model $M = (W, V, \mathcal{S})$ where

- W is a non-empty set of worlds
- $V : \mathit{Atomics} \to \mathcal{P}(W)$ is a valuation, telling us which atomic sentnces are true in which worlds
- $\mathcal{S} : W \to \mathcal{P}(\mathcal{P}(W))$ is a system of spheres, assigning to each world w a set of sets of worlds S_w

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Constraints on the spheres:

1. Each $S \in S_w$ is nonempty
2. S_w is centered on w: $\{w\} \in S_w$
3. S_w is nested: for any $S, S' \in S_w$, $S \subseteq S'$ or $S' \subseteq S$

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$A > C$ is true at w iff

(i) no sphere in S_w contains an A-world, or

(ii) some sphere $S \in S_w$ contains an A-world, and every A-world in S is a C-world

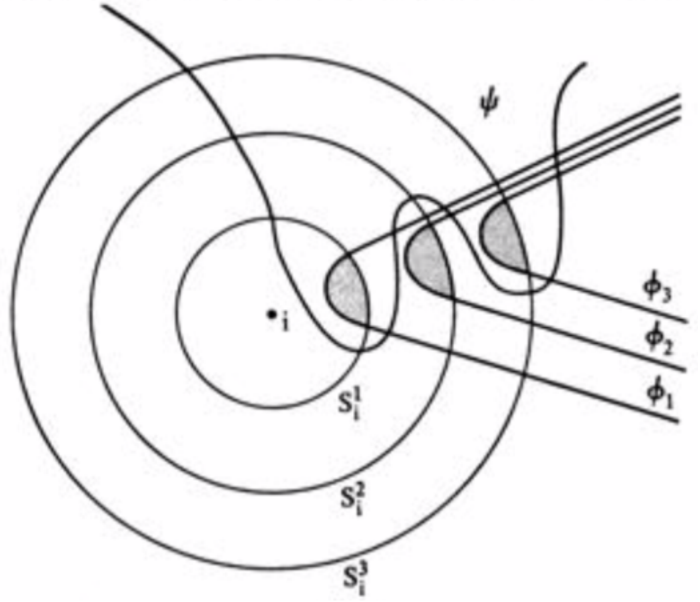

Figure: Lewis's sphere diagrams

Lewis's ordering semantics (Lewis 1981)

Let W be a set. For any $w \in W$ let $\leq_w$ be a reflexive and transitive binary relation over W.

For any sentences A and C and $w \in W$ define that a conditional $A > C$ is true at w (denoted $w \models A > C$) just in case

$$\forall x \models A \ \exists y \models A : y \leq_w x \wedge \forall z \leq_w y, z \models A \supset C,$$

where $A \supset C$ is the material conditional (that is, equivalent to $\neg A \vee C$).

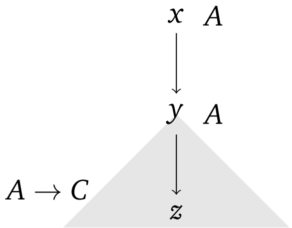

Figure: Illustrating the truth conditions of $A > C$.

(10) a. If this match had been scratched, it would have lit.
b. If this match had been wet and scratched, it would have lit.

more similar ← less similar

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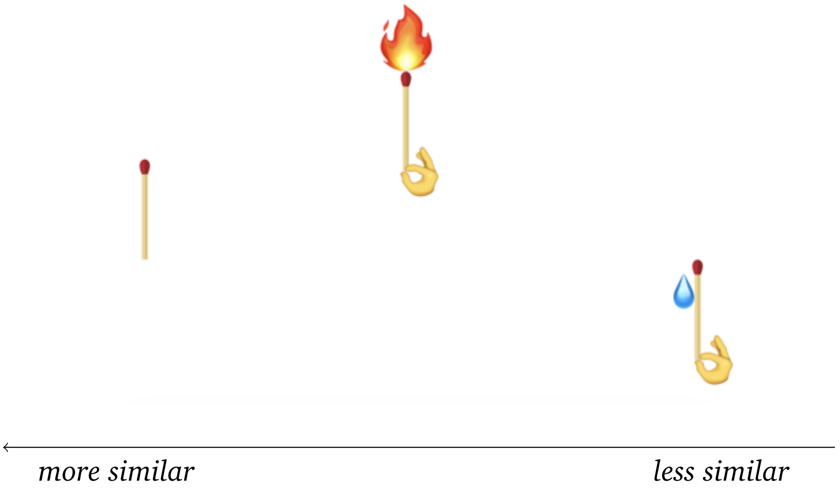

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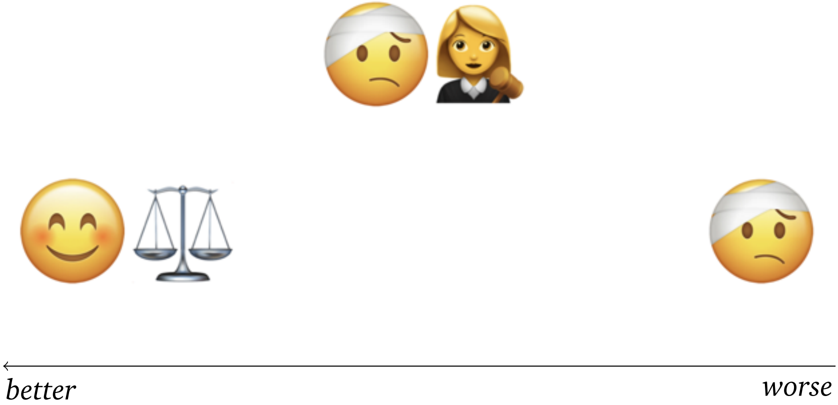
(11) a. Justice must be given to everyone.
b. If someone was unjustly treated, the injustice must be amended.
c. If someone was unjustly treated, the injustice must be rewarded.

Assumptions

1. $\Box A$ is true at w iff A is true at every v accessible from w (Kripke 1959)
2. v is accessible from w iff the rules of w are followed in v
3. *If A, must B* expresses $\Box(A \rightarrow B)$

Problem

$\Box A$ entails $\Box(\neg A \rightarrow B)$ and $\Box(\neg A \rightarrow \neg B)$ (Kratzer 1981, p. 70)


better
worse

Kratzer's premise semantics (Kratzer 1981)

A proposition is a set of worlds. Let f and g be functions, each taking a world and returning a set of propositions.

- f is the *modal base*. It determines the accessibility relation.

  A world w' is accessible from a world w just in case w' makes true every proposition in the modal base at w.

$$wRw' \quad \equiv \quad w' \in \bigcap f(w)$$

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- For bouletic modals (*want*, *desire*), the modal base is doxastic: it only contains worlds compatible with the agent's beliefs at w.

Kratzer's premise semantics (Kratzer 1981)

g is the *ordering source*.

It determines an order over worlds.

One world w' is as good as another w'', relative to $g(w)$, just in case every proposition in $g(w)$ that is true at w'' is true at w'.

$$w' \leq_{g(w)} w'' \quad \equiv \quad \forall p \in g(w) : w'' \in p \rightarrow w' \in p$$

Kratzer's premise semantics (Kratzer 1981)

- g is realistic just in case w makes every proposition in $g(w)$ true: $w \in g(w)$

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  A and $A > C$ imply C

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Kratzer (1981): counterfactuals have a totally realistic ordering source

Lewis (1975) on adverbs of quantification

always, usually, mostly, rarely, sometimes, never...

(12) a. Sometimes, if a cat is happy, it purrs.
b. Never, if a cat is happy, does it purr.
c. Usually, if a cat is happy, does it purr.

Lewis (1975) on adverbs of quantification

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Sometimes/never/usually is this conditional sentence true: "if a cat is happy, it purrs".

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Sometimes/never/usually is this conditional sentence true: "if a cat is happy, it purrs".

But they *can* be parsed as $Q(A)(C)$, where Q is a binary operator

- $\textit{sometimes}(A)(C) = A \cap C \neq \emptyset$
- $\textit{never}(A)(C) = A \subseteq \neg C$
- $\textit{usually}(A)(C) = \#(A \wedge C)$ is greater than $\#(A \wedge \neg C)$

The restrictor view

The history of the conditional is the story of a syntactic mistake. There is no two-place if ... then connective in the logical forms for natural languages. If-clauses are devices for restricting the domains of operators. Whenever there is no explicit operator, we have to posit one.

(Kratzer 1986, p. 656)

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(Kratzer 1986, p. 656)

Kratzer's implementation of the restrictor view

Conditional antecedents add the proposition they express to the modal base.

if A, C is true with respect to w, f, g just in case

C is true with respect to $w, \color{red}{f + A}, g$,

where $f + A(w) = f(w) \cup \{|A|\}$ for any world w, and $|A|$ is the set of worlds where A is true

Kratzer on the restrictor view

quantificational operators, including adverbs of quantification, probability operators, and other modal operators, do not operate over "conditional propositions." The persistent belief that there could be such "conditional propositions" is based on a simple syntactic mistake. If-clauses need to be parsed as adverbial modifiers that restrict operators that might be silent and a distance away. This is what we might call "the restrictor view" of if-clauses.

(Kratzer 2012, p. 107)

ANGELIKA KRATZER

AN INVESTIGATION OF THE LUMPS OF THOUGHT

Contents

Kratzer derives the ordering source from a set of propositions

$$w' \leq_{g(w)} w'' \quad \equiv \quad \forall p \in g(w) : w'' \in p \rightarrow w' \in p$$

Example

Illustration: p and q are true at w.
Consider: "if p were false, q would still be true"

$$g_1(w) = \{p, q\}$$
$$g_2(w) = \{p \cap q\}$$

Kratzer's semantics of *would*-conditionals

There is an intuitive and appealing way of thinking about the truthconditions for counterfactuals. It is an analysis that, in my heart of hearts, I have always believed to be correct...

A "would"-counterfactual is true in a world w iff every way of adding propositions that are true in w to the antecedent while preserving consistency reaches a point where the resulting set of propositions logically implies the consequent.

— Angelika Kratzer (2012, p. 127)

Kratzer's semantics of *would*-conditionals

Definition (Admissible base set)

Let S be the set of situations (i.e. sets of parts of possible worlds). For any world w, an *admissible Base Set* is a subset F_w of $P(S)$ satisfying:

1. Truth: Every proposition in F_w is true at w: $w \in \bigcap F_w$.
2. Persistence: All proposition in F_w is persistent (for all $p \in F_w$ and situations s, s', if $s \in p$ and s is part of s', then $s' \in p$).
3. Cognitive Viability: All $p \in F_w$ are cognitively viable.
4. Non-Redundancy: F_w is not redundant (it does not contain propositions p and q such that $p \neq q$ and $p \cap W \subseteq q \cap W$).
5. Completeness: $\bigcap F_w$ contains all and only worlds that are indistinguishable from w, given the grain set by Cognitive Viability.

Define that proposition p *lumps* proposition q at world w just in case for any situation that is part of w in which p is true, q is true.

Kratzer's semantics of *would*-conditionals

Given an admissible base set and a proposition p, Kratzer defines the *crucial set* $F_{w,p}$ as follows.

Definition (The crucial set)

For any world w, admissible base set F_w, and proposition p, $F_{w,p}$ is the set of all subsets A of $F_w \cup \{p\}$ satisfying the following conditions:

1. A is consistent
2. $p \in A$
3. A is closed under lumping: for all $q \in A$ and $r \in F_w$: if q lumps r in w, then $r \in A$.

Definition (Truth conditions of "would"-counterfactuals)

Given a world w and an admissible Base Set F_w, a "would"-counterfactual with antecedent p and consequent q is true in w iff for every set in $F_{w,p}$ there is a superset in $F_{w,p}$ that logically implies q.

Angelika and the Zebra

Last year, a zebra escaped from the Hamburg zoo. The escape was made possible by a forgetful keeper who forgot to close the door of a compound containing zebras, giraffes, and gazelles. A zebra felt like escaping and took off. The other animals preferred to stay in captivity. Suppose now counterfactually that some other animal had escaped instead. Would it be another zebra? Not necessarily. I think it might have been a giraffe or a gazelle.

(Kratzer 1989, p. 625)

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(Kratzer 1989, p. 625)

Angelika and the Zebra

> *Yet if the similarity theory of counterfactuals were correct, we would expect that, everything else being equal, similarity with the animal that actually escaped should play a role in evaluating this particular piece of counterfactual reasoning. Given that all animals in the compound under consideration had an equal chance of escaping, the most similar worlds to our world in which a different animal escaped are likely to be worlds in which another zebra escaped. That is, on the similarity approach, the counterfactual expressed by [(13)] should be false in our world.*
>
> *(13) If a different animal had escaped instead, it might have been a gazelle.*

(Kratzer 1989, p. 625)

How lumping helps in the zebra case

We have to build the premise sets.

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- If we added "a zebra escaped" we would have to add "John escaped", by closure under lumping

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- But then the set contains both "John escaped" and "if a different animal had escaped instead". But this is inconsistent!

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- We must add the antecedent "a different animal escaped instead"
- If we added "a zebra escaped" we would have to add "John escaped", by closure under lumping
- But then the set contains both "John escaped" and "if a different animal had escaped instead". But this is inconsistent!
- $\therefore$ we cannot add "a zebra escaped" to the premise set

Plan

Example from Quine (1950):

(14) If Bizet and Verdi had been compatriots, they would have both been French.

Example from Quine (1950):

(14) If Bizet and Verdi had been compatriots, they would have both been French.

(15) If Bizet and Verdi had been compatriots, they would have both been Italian.

Conditional Excluded Middle (CEM)

Many argue that $\neg(A \mathrel{\Box\!\!\rightarrow} C)$ is equivalent to $A \mathrel{\Box\!\!\rightarrow} \neg C$ (see e.g. von Fintel 1997)
called ‘negation swap’

Conditional Excluded Middle (CEM)

Many argue that $\neg(A \mathrel{\Box\!\!\rightarrow} C)$ is equivalent to $A \mathrel{\Box\!\!\rightarrow} \neg C$ (see e.g. von Fintel 1997)
called 'negation swap'

Negation swap is equivalent to Conditional Excluded Middle (CEM):

$$(A \mathrel{\Box\!\!\rightarrow} C) \vee (A \mathrel{\Box\!\!\rightarrow} \neg C)$$

for discussion see Lewis (1973b), Stalnaker (1980), Cariani and Santorio (2018), and Mandelkern (2018), among others

Support for CEM

Pair based on Higginbotham (1986):

(16) Everyone will pass if he works hard

(17) No one will fail if he works hard

Failing = not passing

Support for CEM: Higgenbotham's pair

Pair based on Higginbotham (1986):

(18) Everyone will pass if he works hard
$\forall x(\textit{if } Wx, Px)$

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(18) Everyone will pass if he works hard
$\forall x(\mathit{if}\ Wx, Px)$

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$\neg\exists x(\mathit{if}\ Wx, \neg Px)$

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(18) Everyone will pass if he works hard
$\forall x(\textit{if } Wx, Px)$

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$\neg\exists x(\textit{if } Wx, \neg Px)$
$\forall x \neg(\textit{if } Wx, \neg Px)$

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$\forall x \neg(\textit{if}\ Wx, \neg Px)$

CEM ensures the equivalence of Higgenbotham's pair

Support for CEM: Probability (see Mandelkern 2018)

Alice has a fair coin that she didn't flip. She says:

(20) If I had flipped the coin, it would have landed heads.

What is the probability that what Alice said is true?

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What is the probability that what Alice said is true? Intuitively, one half.

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If the probability operator measures the different ways of choosing a single world from the selected worlds, we can predict the intuitive verdict of one half

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